NEHRU COLLEGE OF ENGINEERING AND RESEARCH CENTRE
(NAAC Accredited)
(Approved by AICTE, Affiliated to APJ Abdul Kalam Technological University, Kerala)

DEPARTMENT OF MECHANICAL ENGINEERING

COURSE MATERIALS

ME206 FLUID MACHINERY

VISION OF THE INSTITUTION

To mould true citizens who are millennium leaders and catalysts of change through excellence in education.

MISSION OF THE INSTITUTION

NCERC is committed to transform itself into a center of excellence in Learning and Research in Engineering and Frontier Technology and to impart quality education to mould technically competent citizens with moral integrity, social commitment and ethical values.

We intend to facilitate our students to assimilate the latest technological know-how and to imbibe discipline, culture and spiritually, and to mould them in to technological giants, dedicated research scientists and intellectual leaders of the country who can spread the beams of light and happiness among the poor and the underprivileged.

ABOUT DEPARTMENT

- Established in: 2002
- Course offered : B.Tech in Mechanical Engineering

- Approved by AICTE New Delhi and Accredited by NAAC
- Affiliated to the University of Dr. A P J Abdul Kalam Technological University.

DEPARTMENT VISION

Producing internationally competitive Mechanical Engineers with social responsibility & sustainable employability through viable strategies as well as competent exposure oriented quality education.

DEPARTMENT MISSION

1. Imparting high impact education by providing conductive teaching learning environment.
2. Fostering effective modes of continuous learning process with moral & ethical values.
3. Enhancing leadership qualities with social commitment, professional attitude, unity, team spirit & communication skill.
4. Introducing the present scenario in research & development through collaborative efforts blended with industry & institution.

PROGRAMME EDUCATIONAL OBJECTIVES

PEO1: Graduates shall have strong practical & technical exposures in the field of Mechanical Engineering & will contribute to the society through innovation & enterprise.

PEO2: Graduates will have the demonstrated ability to analyze, formulate & solve design engineering / thermal engineering / materials & manufacturing / design issues & real life problems.

PEO3: Graduates will be capable of pursuing Mechanical Engineering profession with good communication skills, leadership qualities, team spirit & communication skills.

PEO4: Graduates will sustain an appetite for continuous learning by pursuing higher education & research in the allied areas of technology.

PROGRAM OUTCOMES (POS)

Engineering Graduates will be able to:

1. **Engineering knowledge**: Apply the knowledge of mathematics, science, engineering fundamentals, and an engineering specialization to the solution of complex engineering problems.
2. **Problem analysis**: Identify, formulate, review research literature, and analyze complex engineering problems reaching substantiated conclusions using first principles of mathematics, natural sciences, and engineering sciences.
3. **Design/development of solutions**: Design solutions for complex engineering

problems and design system components or processes that meet the specified needs with appropriate consideration for the public health and safety, and the cultural, societal, and environmental considerations.

4. **Conduct investigations of complex problems**: Use research-based knowledge and research methods including design of experiments, analysis and interpretation of data, and synthesis of the information to provide valid conclusions.
5. **Modern tool usage**: Create, select, and apply appropriate techniques, resources, and modern engineering and IT tools including prediction and modeling to complex engineering activities with an understanding of the limitations.
6. **The engineer and society**: Apply reasoning informed by the contextual knowledge to assess societal, health, safety, legal and cultural issues and the consequent responsibilities relevant to the professional engineering practice.
7. **Environment and sustainability**: Understand the impact of the professional engineering solutions in societal and environmental contexts, and demonstrate the knowledge of, and need for sustainable development.
8. **Ethics**: Apply ethical principles and commit to professional ethics and responsibilities and norms of the engineering practice.
9. **Individual and teamwork**: Function effectively as an individual, and as a member or leader in diverse teams, and in multidisciplinary settings.
10. **Communication**: Communicate effectively on complex engineering activities with the engineering community and with society at large, such as, being able to comprehend and write effective reports and design documentation, make effective presentations, and give and receive clear instructions.
11. **Project management and finance**: Demonstrate knowledge and understanding of the engineering and management principles and apply these to one's own work, as a member and leader in a team, to manage projects and in multidisciplinary environments.
12. **Life-long learning**: Recognize the need for, and have the preparation and ability to engage in independent and life-long learning in the broadest context of technological change.

PROGRAM SPECIFIC OUTCOMES (PSO)

PSO1: Students able to apply principles of engineering, basic sciences & analytics including multi variant calculus & higher order partial differential equations..

PSO2: Students able to perform modeling, analyzing, designing & simulating physical systems, components & processes.

PSO3: Students able to work professionally on mechanical systems, thermal systems & production systems

Course No.	Course Name	L-T-P-Credits	Year of Introduction
ME206	**FLUID MACHINERY**	**2-1-0-3**	**2016**

Prerequisite: ME203 Mechanics of Fluids

Course Objectives:
1. To acquire knowledge on hydraulic machines such as pumps and turbines
2. To understand the working of air compressors and do the analysis

Syllabus

Impact of jets, Hydraulic Turbines, Rotary motion of liquids, Rotodynamic pumps, Positive displacement pumps, , Compressors

Expected outcome: At the end of the course the students will be able to
1. Discuss the characteristics of centrifugal pump and reciprocating pumps
2. Calculate forces and work done by a jet on fixed or moving plate and curved plates
3. Know the working of turbines and select the type of turbine for an application.
4. Do the analysis of air compressors and select the suitable one for a specific application

Text Books:

1. Som, Introduction to Fluid Mechanics and Fluid Machines ,McGraw Hill Education India 2011
2. Bansal R. K., A Textbook of Fluid Mechanics and Hydraulic Machines, Laxmi Publications,2005.

Reference Books:

1. Cengel Y. A. and J. M. Cimbala, Fluid Mechanics, Tata McGraw Hill, 2013
2. Yahya S. M, Fans, Blower and Compressor, Tata McGraw Hill, 2005.
3. Shepherd D. G, Principles of Turbo Machinery, Macmillan, 1969.
4. Stepanoff A. J, Centrifugal and Axial Flow Pumps, John Wiley & Sons, 1991.
5. Rajput R. K, Fluid Mechanics and Hydraulic Machines, S. Chand & Co.,2006.
6. Subramanya, Fluid mechanics and hydraulic machines, 1e McGraw Hill Education India,2010

Course Plan			
Module	**Contents**	**Hours**	**Sem. Exam Marks**
I	Impact of jets: Introduction to hydrodynamic thrust of jet on a fixed and moving surface (flat and curve),– Series of vanes - work done and efficiency Hydraulic Turbines : Impulse and Reaction Turbines – Degree of reaction – Pelton Wheel – Constructional features - Velocity triangles – Euler's equation – Speed ratio, jet ratio and work done , losses and efficiencies, design of Pelton wheel – Inward and outward flow reaction turbines- Francis Turbine – Constructional features – Velocity triangles, work done and efficiencies.	7	15%
II	Axial flow turbine (Kaplan) Constructional features – Velocity triangles- work done and efficiencies – Characteristic curves of turbines – theory of draft tubes – surge tanks – Cavitation in turbines – Governing of turbines – Specific speed of turbine , Type Number– Characteristic curves, scale Laws – Unit speed – Unit discharge and unit power.	7	15%
FIRST INTERNAL EXAM			
III	Rotary motion of liquids – free, forced and spiral vortex flows Rotodynamic pumps- centrifugal pump impeller types,-velocity triangles-manometric head- work, efficiency and losses, H-Q characteristic, typical flow system characteristics, operating point of a pump. Cavitation in centrifugal pumps- NPSH required and available- Type number-Pumps in series and parallel operations. Performance characteristics- Specific speed-Shape numbers – Impeller shapes based on shape numbers.	7	15%
IV	Positive displacement pumps- reciprocating pump – Single acting and double acting- slip, negative slip and work required and efficiency- indicator diagram- acceleration head - effect of acceleration and friction on indicator diagram – speed calculation- Air vessels and their purposes, saving in work done to air vessels multi cylinder pumps. Multistage pumps-selection of pumps-pumping devices-hydraulic ram, Accumulator, Intensifier, Jet pumps, gear pumps, vane pump and lobe pump.	7	15%
SECOND INTERNAL EXAM			
V	Compressors: classification of compressors, reciprocating compressor-single stage compressor, equation for work with and without clearance volume, efficiencies, multistage compressor, intercooler, free air delivered (FAD)	7	20%
VI	Centrifugal compressor-working, velocity diagram, work done, power required, width of blades of impeller and diffuser, isentropic efficiency, slip factor and pressure coefficient, surging and chocking. Axial flow compressors:- working, velocity diagram, degree of reaction, performance. Roots blower, vane compressor, screw compressor.	7	20%
END SEMESTER EXAM			

Question Paper Pattern

Total marks: 100, Time: 3 hrs

The question paper should consist of three parts

Part A

4 questions uniformly covering modules I and II. Each question carries 10 marks

Students will have to answer any three questions out of 4 (3X10 marks =30 marks)

Part B

4 questions uniformly covering modules III and IV. Each question carries 10 marks

Students will have to answer any three questions out of 4 (3X10 marks =30 marks)

Part C

6 questions uniformly covering modules V and VI. Each question carries 10 marks

Students will have to answer any four questions out of 6 (4X10 marks =40 marks)

Note: In all parts, each question can have a maximum of four sub questions, if needed.

Course Outcome

CO No.	Course Outcome
C206.1	Analyze the impact of jets, impulse turbines and radial flow reaction turbines
C206.2	Analyze the axial flow turbine and characteristics of turbines
C206.3	Analyze the principle of operation, and performance characteristics of Centrifugal pumps.
C206.4	Analyze the working principles and performance of reciprocating pumps and summarize the various hydraulic devices
C206.5	Analyze the working principles and performance of reciprocating compressors
C206.6	Analyze the working principles and performance of centrifugal and axial flow compressors

CO-PO-PSO Mapping

CO Vs PO														
ME206														
COURSE COUTCOME	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12	PSO1	PSO2	PSO3
C206.1	3	2	2	-	-	1	-	-	-	-	-	2	2	3	2
C206.2	3	2	2	-	2	1	-	-	-	-	-	2	2	3	2
C206.3	3	2	2	-	2	1	-	-	-	-	-	2	2	3	2
C206.4	3	2	2	-	2	1	-	-	-	-	-	2	2	3	2
C206.5	3	2	2	-	2	1	-	-	-	-	-	2	2	3	2
C206.6	3	2	2	-	2	1	-	-	-	-	-	2	2	3	-

QUESTION BANK

MODULE 1			
SL NO	**QUESTIONS**	**CO**	KL
1	Give example for a low head, medium head and high head turbine.	CO1	K2
2	What is impulse turbine? Give example	CO1	K2
3	What is reaction turbine? Give example	CO1	K2
4	At a location for a hydroelectric plant, the head available (net) was 335 m. The power availability with an overall efficiency of 86% was 15500 kW. The unit is proposed to run at 500 rpm. Assume Cv = 0.98, φ = 0.46, Blade velocity coefficient is 0.9. If the bucket outlet angle proposed is 165° check for the validity of the assumed efficiency.	CO1	K4
5	The jet velocity in a pelton turbine is 65 m/s. The peripheral velocity of the runner is 25 m/s. The jet is deflected by 160° by the bucket. Determine the power developed and hydraulic efficiency of the turbine for a flow rate of 0.9 m3 /s. The blade friction coefficient is 0.9	CO1	K4
6	A Francis turbine works under a head of 120 m. The outer diameter and width are 2 m and 0.16 m. The inner diameter and width are 1.2 m and 0.27 m. The flow velocity at inlet is 8.1 m/s. The whirl velocity at outlet is zero. The outlet blade angle is 16°. Assume η_H = 90%. Determine, power, speed and blade angle at inlet and guide blade angle	CO1	K4
7	In an inward flow reaction turbine the working head is 10 m. The guide vane outlet angle is 20°. The blade inlet angle is 120°. Determine the hydraulic efficiency assuming zero whirl at exit and constant flow velocity. Assume no losses other than at exit	CO1	K4
8	Define hydraulic efficiency	CO1	K2
9	Define mechanical efficiency	CO1	K2
10	Define volumetric efficiency	CO1	K2

MODULE 2			
SL NO	**QUESTIONS**	**CO**	**KL**
1	A Kaplan turbine delivers 30 MW and runs at 175 rpm. Overall efficiency is 85% and hydraulic efficiency is 91%. The tip diameter 5 m and the hub diameter is 2 m. determine the head and the blade angles at the mid radius. The flow rate is 140 m3 /s.	CO2	K4
2	A Kaplan turbine delivers 10 MW under a head of 25 m. The hub and tip diameters are 1.2 m and 3 m. Hydraulic and overall efficiencies are 0.90 and 0.85. If both velocity triangles are	CO2	K4

	right angled triangles, determine the speed, guide blade outlet angle and blade outlet angle.		
3	What is draft tube, Describe with neat sketches two different types of drat tubes.	CO2	K4
4	Define specific speed of a turbine	CO2	K2
5	Derive an expression for specific speed, what is the significance of specific speed.	CO2	K4
6	What are unit quantities, Define the unit quantities for a turbine	CO2	K2
7	Obtain an expression for unit speed, unit discharge, and unit power for a turbine.	CO2	K4
8	What do you understand by characteristics curves of a turbine, Name the important types.	CO2	K3
9	Define the term governing of a turbine. Describe with neat sketch the working of an oil pressure governor.	CO2	K4
10	Explain the difference between Kaplan and Propeller turbines.	CO2	K3

MODULE 3			
SL NO	**QUESTIONS**	**CO**	**KL**
1	Define a centrifugal pump; explain the working of a single stage centrifugal pump with sketches.	CO3	K2
2	Differentiate between the volute casing and vortex casing for the centrifugal pump.	CO3	K4
3	Obtain an expression for the work done by impeller of a centrifugal pump on water per unit weight of water	CO3	K4
4	Define the terms- suction head, delivery head, static head and monometric head	CO3	K2
5	Define the terms- manometric efficiency, mechanical efficiency and overall efficiency.	CO3	K2
6	Define specific speed of a centrifugal pump, derive an expression for the same.	CO3	K4
7	What do you understand by characteristic curve of a pump? What is the significance of these curves?	CO3	K3
8	A centrifugal pump running at 900 rpm has an impeller diameter of 500 mm and eye diameter of 200 mm. The blade angle at outlet is 35° with the tangent. Determine assuming zero whirl at inlet, the inlet blade angle. Also calculate the absolute velocity at outlet and its angle with the tangent. The flow velocity is constant at 3 m/s. Also calculate the manometric head.	CO3	K4
9	What is meant by Priming?	CO3	K2
10	The dimensionless specific speed of a centrifugal pump is	CO3	K4

	0.06. Static head is 30 m. Flow rate is 50 l/s. The suction and delivery pipes are each of 15 cm diameter. The friction factor is 0.02. Total length is 55 m other losses equal 4 times the velocity head in the pipe. The vanes are forward curved at 120°. The width is one tenth of the diameter. There is a 6% reduction in flow area due to the blade thickness. The manometric efficiency is 80%. Determine the impeller diameter.		

MODULE 4			
SL NO	**QUESTIONS**	**CO**	**KL**
1	What is a reciprocating pump? Describe the principle and working of a reciprocating pump with a neat sketch.	CO4	K3
2	Define slip, percentage of slip and negative slip of a reciprocating pump.	CO4	K2
3	Define indicator diagram, how will you prove that the area of indicator diagram is proportional to the work done by the reciprocating pump.	CO4	K4
4	Draw an indicator diagram; consider the effect of acceleration and friction in suction and delivery pipes. Find an expression for the work done / sec in case of single acting reciprocating pump.	CO4	K4
5	What is an air vessel; describe the function of the air vessel for reciprocating pumps.	CO4	K2
6	Show from first principle that the work saved against friction in the delivery pipe of a single acting reciprocating pump by fitting an air vessel is 84.8% while for a double acting reciprocating pump the work saved is only 39.2%.	CO4	K3
7	What is hydraulic accumulator; explain its principle and working	CO4	K2
8	Explain the working of hydraulic intensifier with neat sketches	CO4	K2
9	Explain the working of the following devices in detail, Jet pump, Lobe pump, Vane pump and screw pump.	CO4	K4
10	A single acting reciprocating pump, running at 60 rpm delivers 0.53m3 of water per minute. The diameter of the piston is 200 mm and stroke length 300 mm. The suction and delivery heads are 4 m and 12 m respectively. Determine Theoretical discharge, Co-efficient of discharge, % slip of the pump and Power required running the pump.	CO4	K3

MODULE 5			
SL NO	**QUESTIONS**	**CO**	**KL**
1	What is the application of compressed air?	CO5	K2
2	Write a short note on double acting air compressor.	CO5	K3
3	A single acting reciprocating pump has a piston diameter 100mm and stroke length 200 mm. The length and diameter of the suction pipe are 6.5 m and 50 mm respectively. If the suction lift of the pump is 3.2 m and separation occurs when the pressure in the pump falls below 2.5 m of water absolute. The barometer reads 763 mm of mercury. Find the maximum speed at which the pump can run without separation in the suction pipe.	CO5	K2
4	A single stage reciprocating air compressor is compressing 2 Kg of air per minute at 1 bar 20 C and delivers it at 7 bar. Assume compression process follows the law PV1.3 =C. Calculate indicated power input to compressor. Neglect clearance.	CO5	K3
5	Derive the condition for minimum work required for a 2-stage reciprocating air compressor.	CO5	K2
6	What is the volumetric efficiency of a reciprocating air compressor?	CO5	K4
7	What is isothermal efficiency of a reciprocating air compressor?	CO5	K2
8	Define free air delivered (FAD)	CO5	K3

MODULE 6			
SL NO	**QUESTIONS**	**CO**	**KL**
1	What are the main components of a centrifugal compressor?	CO6	K2
2	Define slip factor	CO6	K3
3	What is work factor?	CO6	K2
4	What is meant by stalling?	CO6	K4
5	Discuss the merits and demerits of a centrifugal compressor over axial flow compressor.	CO6	K2
6	Derive the expression for width of impeller blade for centrifugal compressor.	CO6	K2
7	Write a short note on the following devices. a. Lobe compressor b. Vane compressor C. Screw compressor. d. Roots blower.	CO6	K4
8	What is Degree of Reaction?	CO6	K3

APPENDIX 1	
CONTENT BEYOND THE SYLLABUS	
SLNO.	**WEB SOURCE REFERENCES**
1	http://nptel.ac.in/courses/Webcourse-contents/IIT-KANPUR/machine/ui/Course_home-7.htm
2	http://nptel.ac.in/courses/112105182/9
3	http://www.slideshare.net/ArchieSecorata/fluid-mechanicsfundamentals-and-applications-by-cengel-cimbala-3rd-c2014-txtbk
4	https://www.youtube.com/watch?v=RBVgwpYUp18
5	https://www.youtube.com/watch?v=KqfYobOYRTc

FLUID MACHINERY

MODULE - 1, 2 → water turbines → Pelton wheel, Francis turbine, Kaplan

MODULE - 3, 4 → Pumps → Reciprocating pump, centrifugal pump

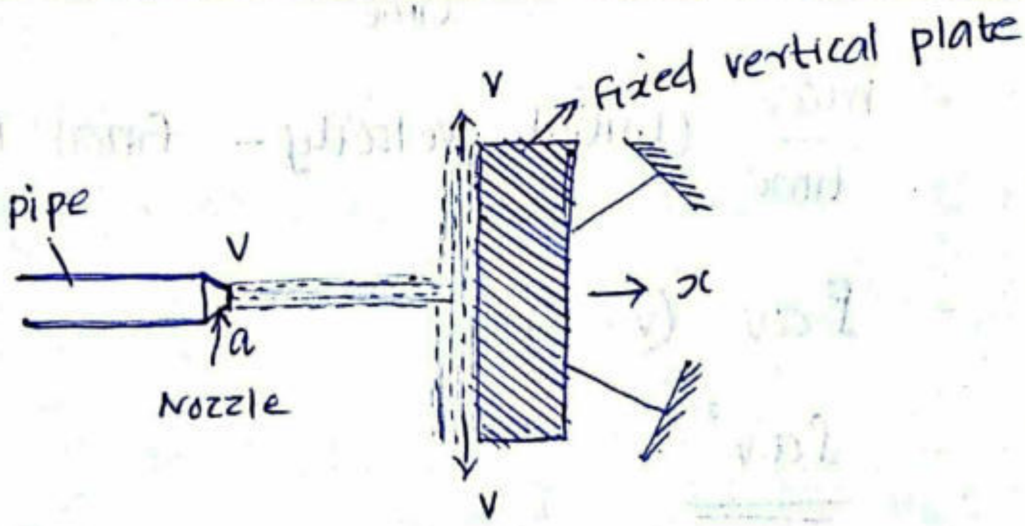
MODULE - 5, 6 → Compressors → Reciprocating compressors, centrifugal compressor

MODULE - 1

Impact of Jet

case - 1

Impact of jet on fixed vertical flat plate



Consider a jet of water coming out from the nozzle, strikes a flat vertical fixed plate

let V = velocity of jet

a = area of cross section of jet

The jet after striking the plate will move along the plate, ie; the jet will deflected through 90°. Hence the component of velocity of jet in the direction of jet after striking will be zero. The force excerted by the jet on the plate in x - direction;

F_x = Rate of change of momentum in x- direction

$$= \frac{\text{Intial momentum} - \text{Final momentum}}{\text{time}}$$

$$= \frac{(\text{mass} \times \text{initial velocity}) - (\text{mass} \times \text{final velocity})}{\text{time}}$$

$$= \frac{\text{mass}}{\text{time}} (\text{Initial velocity} - \text{final velocity})$$

$$= \rho a v (v - 0)$$

$$= \underline{\underline{\rho a v^2}}$$

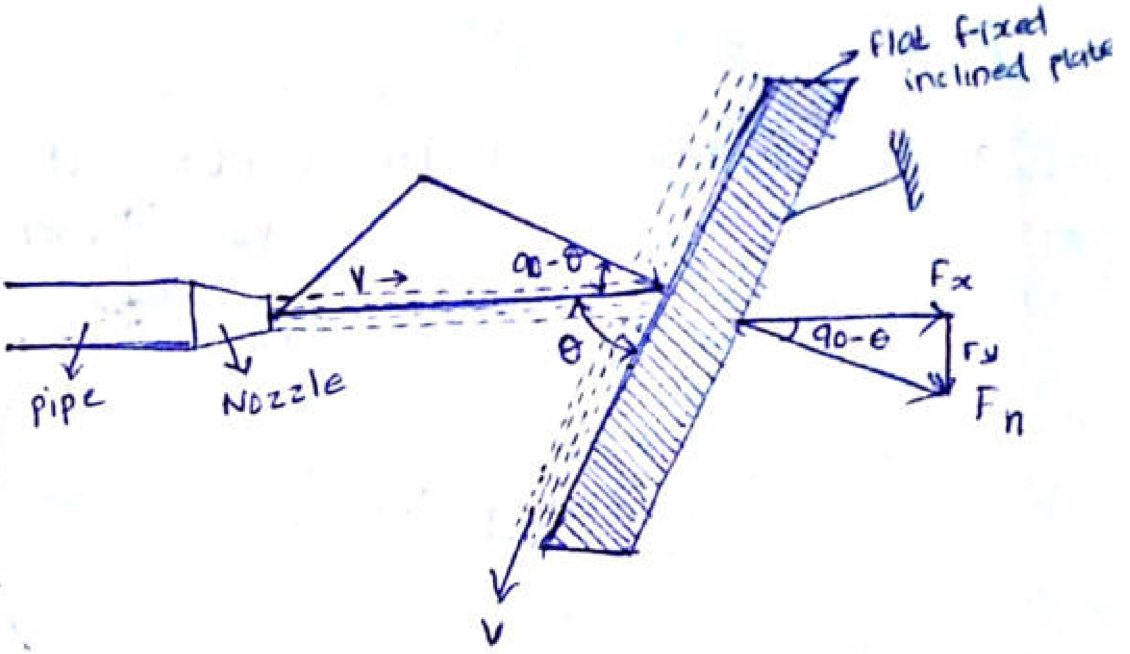
$$\rho = \frac{m}{V} = \frac{m/\text{sec}}{V/\text{sec}} = \frac{m/\text{sec}}{Q}$$

$$m/\text{sec} = \rho \cdot Q = \rho a v$$

05-01-2018

Case - 2

Force exerted by a jet on a flat fixed inclined plate



$$F_n = \frac{mass}{sec} (\text{initial velocity} - \text{final velocity})$$

$$= \rho a v \, (v \sin\theta - 0)$$

$$\boxed{F_n = \rho a v^2 \sin\theta}$$

$$F_x = F_n \cos(90-\theta) = F_n \sin\theta = \rho a v^2 \sin^2\theta$$

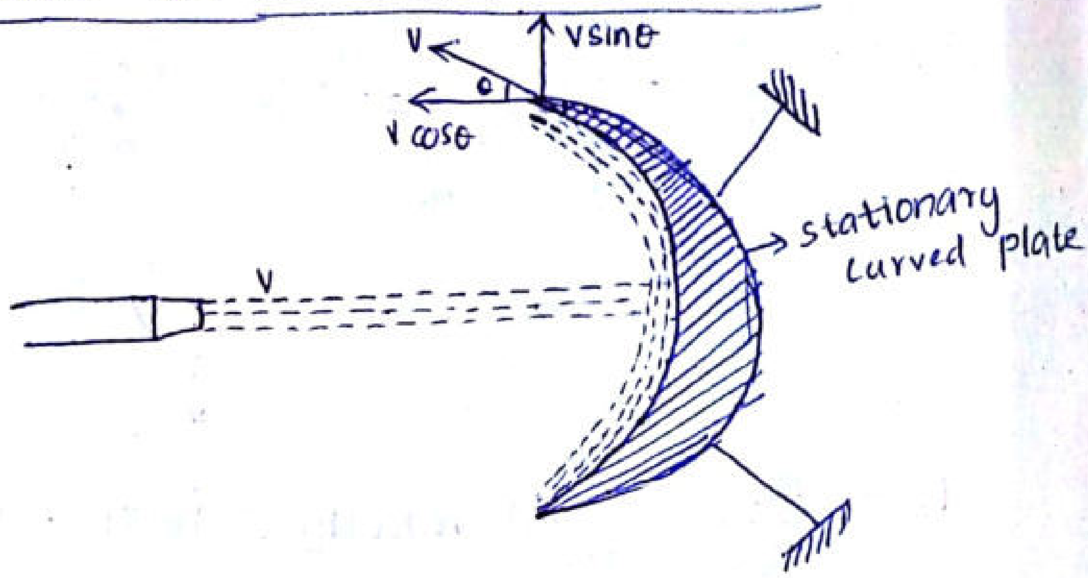
$$F_y = F_n \sin(90-\theta) = F_n \cos\theta = \rho a v^2 \sin\theta \cos\theta$$

$$\boxed{\begin{aligned} F_x &= \rho a v^2 \sin^2\theta \\ F_y &= \rho a v^2 \sin\theta \cos\theta \end{aligned}}$$

case - 3

Force excerted by a jet on a stationary curved plate

a) Jet strikes at the centre of plate.



$$F_x = \frac{\text{mass}}{\text{sec}} [\text{initial velocity} - \text{Final velocity}]_{\text{in } x\text{-direction}}$$

$$= \rho a v \, [v - - v\cos\theta]$$

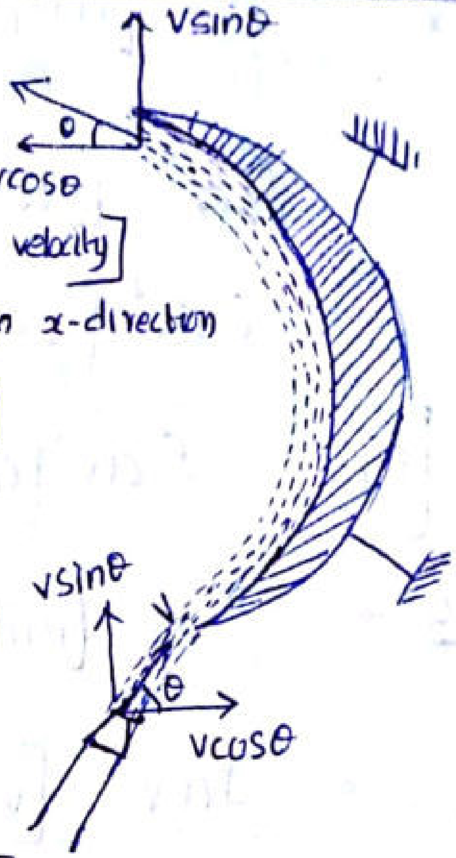
$$\boxed{F_x = \rho a v^2 [1 + \cos\theta]}$$

$$F_y = \frac{\text{mass}}{\text{sec}} [\text{initial velocity} - \text{final velocity}]_{\text{in } y\text{-direction}}$$

$$= \rho a v \, [0 - v\sin\theta]$$

$$\boxed{F_y = - \rho a v^2 \sin\theta}$$

b) Jet strikes at one end of plate and plate is symmetrical



$$F_x = \frac{mass}{sec}\left[\text{initial velocity} - \text{Final velocity}\right] \text{ in } x\text{-direction}$$

$$= \rho a v\left[v\cos\theta - - v\cos\theta\right]$$

$$= \rho a v^2 \cos\theta\,[1+1]$$

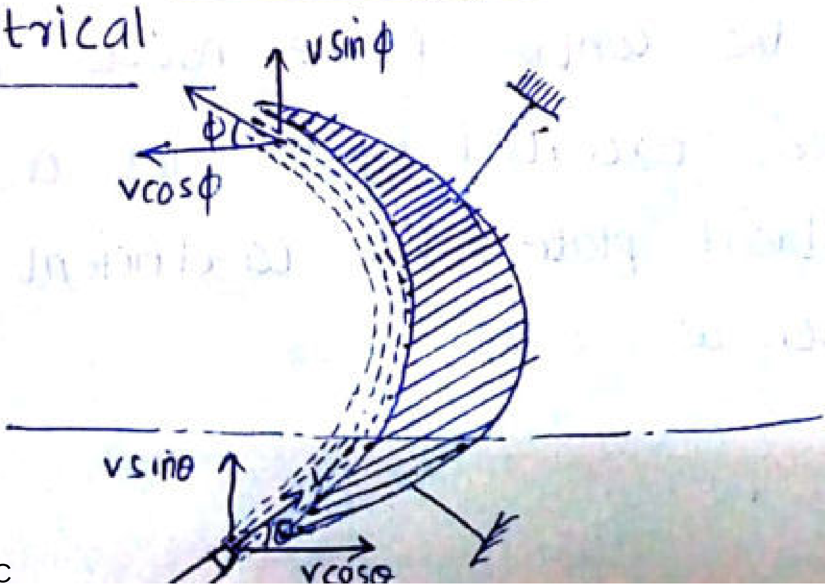
$$\boxed{F_x = 2\rho a v^2 \cos\theta}$$

$$F_y = \frac{mass}{sec}\left[\text{initial velocity} - \text{Final velocity}\right] \text{ in } y \text{ direction}$$

$$= \rho a v\left[v\sin\theta - v\sin\theta\right]$$

$$\boxed{F_y = 0}$$

c) Jet strikes at one end of plate and plate is unsymmetrical



$$F_x = \frac{\text{mass}}{\text{sec}}\left[\text{initial velocity} - \text{Final velocity}\right]_{\text{in } x\text{-direction}}$$

$$= \rho a v\left[v\cos\theta - -v\cos\phi\right]$$

$$= \rho a v^2\left[\cos\theta + \cos\phi\right]$$

$$\boxed{F_x = \rho a v^2\left[\cos\theta + \cos\phi\right]}$$

$$F_y = \frac{\text{mass}}{\text{sec}}\left[\text{initial velocity} - \text{Final velocity}\right]_{\text{in } y\text{-direction}}$$

$$= \rho a v\left[v\sin\theta - v\sin\phi\right]$$

$$= \rho a v^2\left[\sin\theta - \sin\phi\right]$$

$$\boxed{F_y = \rho a v^2(\sin\theta - \sin\phi)}$$

Q) Water is flowing through a pipe at the end of which a nozzle is fitted. the diameter of the nozzle is 100mm and the head of water at the centre of the nozzle is 100m. Find the force excerted by the jet of water on a fixed vertical plate. The co-efficient of velocity is given as 0.95

given

diameter of nozzle, $d = 100mm = 0.1\ m$

area of cross section, $a = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times 0.1^2 = 7.85 \times 10^{-3}\ m^2$

head of water, $H = 100m$.

co-efficient of velocity, $C_v = 0.95$

$V_{act} = C_v \sqrt{2gH}$

$= 0.95 \times \sqrt{2 \times 9.81 \times 100}$

$= 42.07\ m/s$

ρ of water $= 1000\ kg/m^3$ //

$C_v = \frac{V_{act}}{V_{theor}}$

$\cancel{V_{ac}}$

$V_{theoretical} = \frac{V_{act}}{C_v} = \frac{42.0755}{0.95} = 44.29\ m/s$

$F_x = \rho a v^2$

$= 1000 \times 7.85 \times 10^{-3} \times 44.29^2$

$= 13893.59\ N$

Q) A Jet of water of diameter 50mm strikes a fixed plate in such a way that the angle between the plate and jet is 30°. The force exerted in the direction of jet is 1471.5 N. Determine the rate of flow of water

given

$d = 50mm = 0.05 m$

$\theta = 30°$

$F_x = 1471.5 N$

$a = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times 0.05^2 = 1.963 \times 10^{-3} m^2$

$F_x = \rho a v^2 \sin^2\theta$

$$v^2 = \frac{F_x}{\rho a \sin^2\theta}$$

$$= \frac{1471.5}{1000 \times 1.963 \times 10^{-3} \times \sin^2 30}$$

$v^2 = 2998.471$

$v = 54.75 \ m/s$

$$Q = a \cdot v$$
$$= 1.963 \times 10^{-3} \times 54.75$$
$$= \underline{0.107} \ m^3/s$$

06-01-2018

1. Flat vertical plate $\quad F_x = \rho a v^2$
2. Flat inclined plate $\quad F_n = \rho a v^2 \sin\theta$

$$F_x = \rho a v^2 \sin^2\theta$$
$$F_y = \rho a v^2 \sin\theta \cos\theta$$

3. curved plate

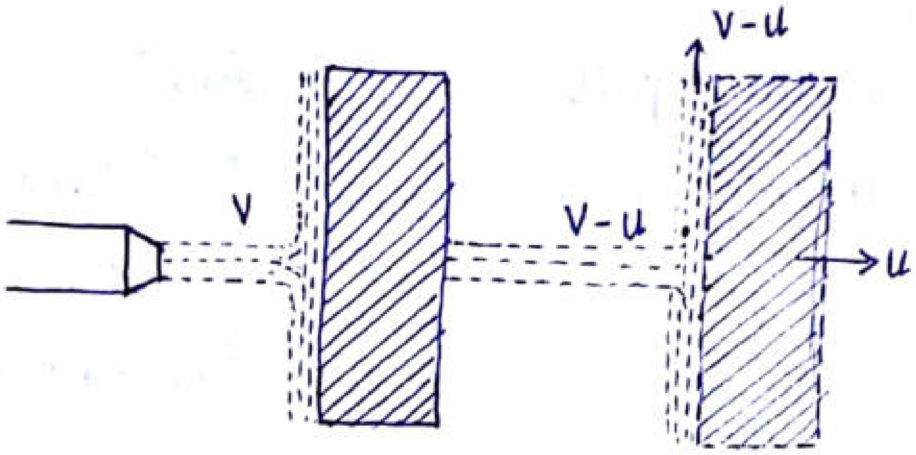
a) jet strikes at centre
$$F_x = \rho a v^2 (1 + \cos\theta)$$
$$F_y = -\rho a v^2 \sin\theta$$

b) jet strikes at one end
$$F_x = 2 \rho a v^2 \cos\theta$$
$$F_y = 0$$

c) Plates is unsymmetrical
$$F_x = \rho a v^2 (\cos\theta + \cos\phi)$$
$$F_y = \rho a v^2 (\sin\theta - \sin\phi)$$

Force exerted by jet on moving plate

1. Force exerted by jet on a vertical flat moving plate in the direction of jet



Relative velocity of jet before striking = $V-u$

Mass of fluid striking /sec = mass/sec = $\rho a(V-u)$

$$F_x = \frac{\text{mass}}{\text{sec}}\left[\text{initial velocity} - \text{Final velocity}\right]_{\text{in x-direction}}$$

$$F_x = \rho a(V-u)\left[(V-u)-0\right]$$

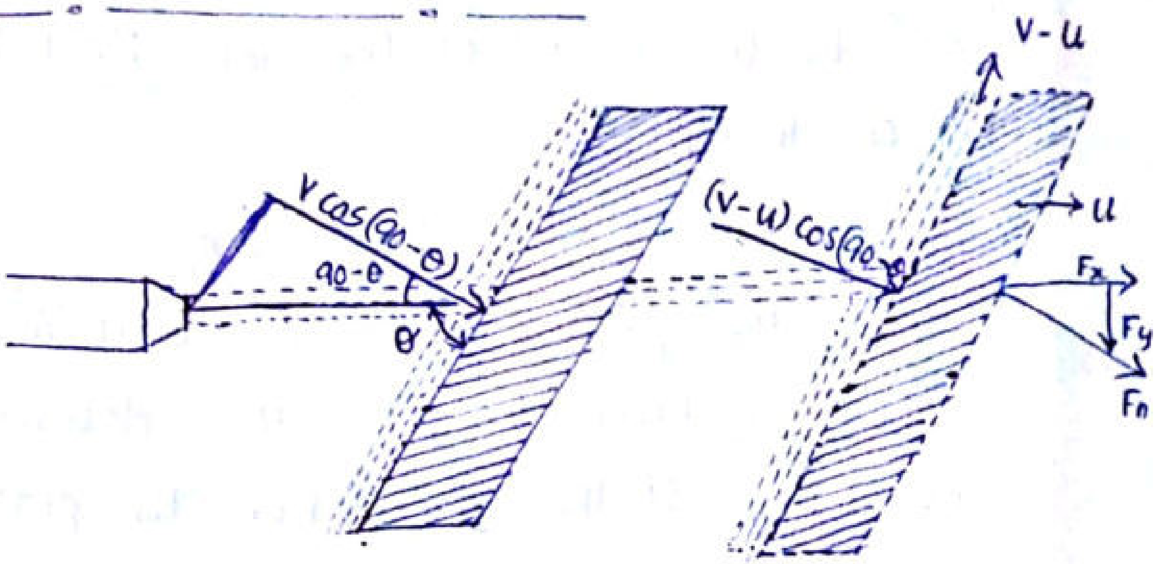
$$F_x = \rho a(V-u)^2$$

$$\text{workdone}/\text{sec} = F_x \times u$$

$$= \rho a(V-u)^2 \times u \quad \text{Nm/s or watts}$$

$$\eta = \frac{\text{output}}{\text{input}} = \frac{\text{workdone}/\text{sec}}{\text{K.E of jet}} = \frac{F_x \times u}{\frac{1}{2}mV^2} = \frac{F_x \times u}{\frac{1}{2}(\rho a V)V^2}$$

2. Force exerted by jet on a inclined flat moving plate in the direction of jet



Relative velocity of jet before striking = $V-u$

mass of fluid striking /sec = mass/sec = $\rho a (V-u)$

$$F_n = \frac{mass}{sec}\left[\text{initial velocity} - \text{Final velocity}\right]_{\text{in normal direction}}$$

$$= \rho a (V-u)\left[(V-u)\sin\theta - 0\right]$$

$$= \rho a (V-u)^2 \sin\theta$$

$F_x = F_n \cos(90-\theta) = F_n \sin\theta = \rho a (V-u)^2 \sin^2\theta$

$F_y = F_n \sin(90-\theta) = F_n \cos\theta = \rho a (V-u)^2 \sin\theta \cos\theta$

work done/sec $= F_x \times u$

$$\eta = \frac{\text{output}}{\text{input}} = \frac{\text{workdone/sec}}{\text{K.E of jet}} = \frac{F_x \times u}{\frac{1}{2} m V^2}$$

$$= \frac{F_x \times u}{\frac{1}{2}(\rho a V) V^2}$$

Q) A 7.5 cm diameter jet having a velocity of 30 m/s strikes a flat plate, the normal of which is inclined at 45° to the axis of the jet. Find the normal pressure on the plate

(a) when the plate is stationery

(b) when the plate is moving with a velocity of 15 m/s and away from the jet. also determine power & efficiency of the jet when the plate is moving

given

$d = 7.5\,cm = 0.075\,m$

$a = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times 0.075^2 = 4.417 \times 10^{-3}\,m^2$

$V = 30\,m/s$

$\theta = 45°$

$u = 15\,m/s$

(a) $F_n = \rho a v^2 \sin\theta$ (when the plate is stationary)

$= 1000 \times 4.417 \times 10^{-3} \times 30^2 \times \sin 45$

$= 2810.961\,N$

(b) when the plate is moving

$F_n = \rho a (V-u)^2 \sin\theta$

$= 1000 \times 4.417 \times 10^{-3} \times (30-15)^2 \sin 45$

$= 702.740 \ N$

$$\text{power} = \frac{\text{Work done}}{\text{sec}} = F_x \cdot u$$

$$\text{Power} = \rho a (v-u)^2 \sin^2\theta \times u$$

$$= 1000 \times 4.417 \times 10^{-3} \times (30-15)^2 \sin^2 45 \times 15$$

$$= 7453.68 \ W$$

$$\cancel{\text{Power}} = \cancel{F_x \times u}$$

$$\cancel{7453.68 \times 15}$$

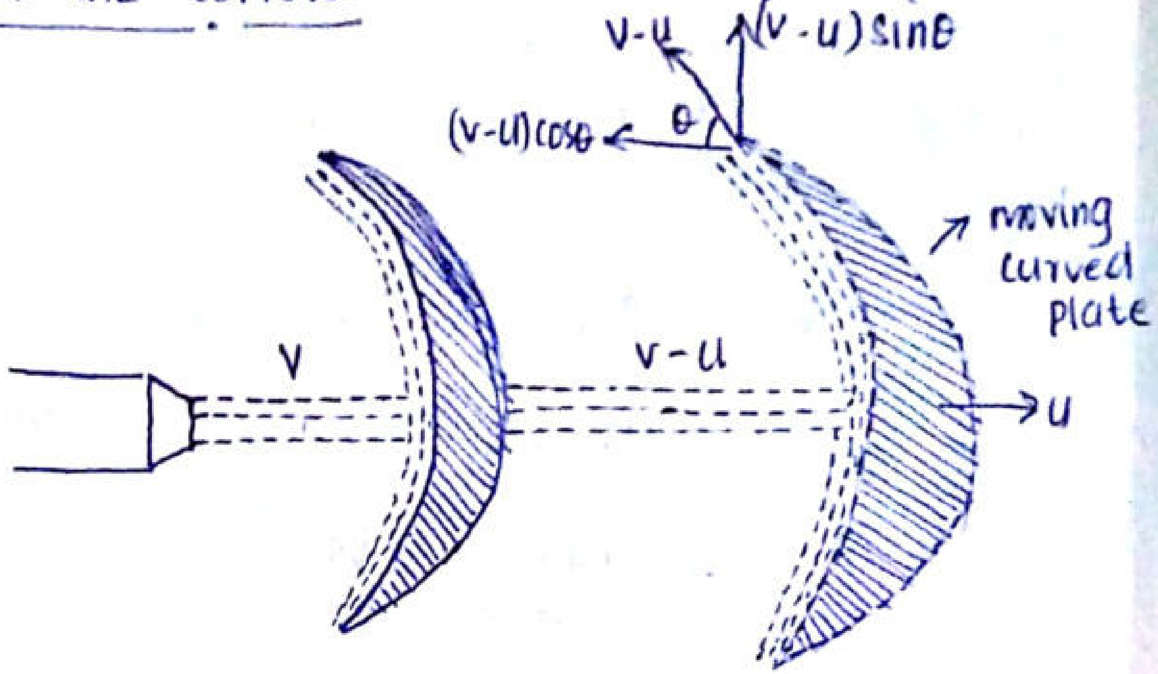
$$\cancel{W}$$

$$\text{Efficiency, } \eta = \frac{F_x \times u}{\frac{1}{2}(\rho a v) v^2}$$

$$= \frac{7453.68}{\frac{1}{2} \times 1000 \times 4.417 \times 10^{-3} \times 30 \times 30}$$

$$= 0.124 = 12.4\%$$

3. Force exerted on a moving curved plate and jet strikes at the centre.



$$F_x = \frac{\text{mass}}{\text{sec}}\left[\text{initial velocity} - \text{Final velocity}\right]$$

$$= \rho a (V-u)\left[(V-u) - -(V-u)\cos\theta\right]$$

$$\boxed{F_x = \rho a (V-u)^2 (1+\cos\theta)}$$

$$F_y = \frac{\text{mass}}{\text{sec}}\left[\text{initial velocity} - \text{Final velocity}\right]$$

$$= \rho a (V-u)\left[0 - (V-u)\sin\theta\right]$$

$$\boxed{F_y = -\rho a (V-u)^2 \sin\theta}$$

$$\text{Power} = \text{workdone/sec} = F_x \cdot u$$

$$\eta = \frac{\text{workdone/sec}}{\frac{1}{2}(\rho a V) V^2}$$

Q) A Jet of water of diameter 7.5cm strikes a curved plate at its center with velocity of 20m/s the curved plate is moving with a velocity of 8m/s in the direction of jet. The jet is deflected through an angle of 165°. assuming the plate is smooth Find:

(i) Force excerted on the plate in the direction of jet

(ii) Power of jet

(iii) Efficiency of jet

given

$d = 7.5\,cm = 0.075\,m$

$a = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times 0.075^2 = 4.417 \times 10^{-3}\,m^2$

$V = 20\,m/s$

$u = 8\,m/s$

$\theta = 180 - 165 = 15°$

(i) $F_x = \rho a (V-u)^2 (1 + \cos\theta)$

$= 1000 \times 4.417 \times 10^{-3} \times (20-8)^2 (1 + \cos 15)$

$= 1250.423\,N$

(ii) Power $= F_x \times u = 1250.423 \times 8 = 10003.38\,W$

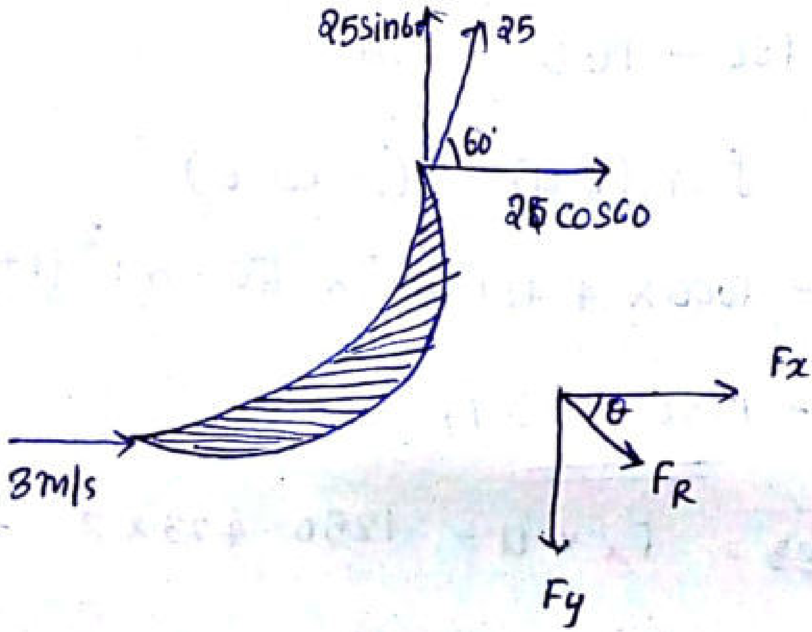
(iii) $\eta = \dfrac{F_x \cdot u}{\frac{1}{2}(\rho a v) \cdot v^2}$

$= \dfrac{1000\, 3 \cdot 38}{\frac{1}{2} \times 1000 \times 4 \cdot 417 \times 10^{-3} \times 20^{3}}$

$= 0 \cdot 566$

$= \underline{\underline{56 \cdot 6\,\%}}$

Q) A Jet of water from a nozzle is deflected through 60° from its original direction by a curved plate which it enters tangentially without shock with a velocity of 30 m/s and leaves with a mean velocity of 25 m/s. If the discharge from the nozzle is 0.8 kg/sec calculate the magnitude and direction of the resultant force on the vane. If the vane is stationary.



given

mass/sec = 0.8 kg/sec

initial velocity - 30 m/s.

F_x = mass/sec [initial velocity − Final velocity]

$= 0.8 \times [30 - 25\cos 60]$

$= 14\ N$

F_y = mass/sec $[0 - 25 \sin 60]$

$= 0.8\ [0 - 25 \sin 60]$

$= -17.32\ N$

~~direction~~, $F_R = \sqrt{F_x^2 + F_y^2}$

$= \sqrt{14^2 + (-17.32)^2} = 22.27\ N$

$\tan\theta = \dfrac{F_y}{F_x}$

$\theta = \tan^{-1}\left(\dfrac{-17.32}{14}\right) = -51.05^\circ$

$\theta = 51.05^\circ$ (anticlockwise direction)

1. Flat vertical moving plate $F_x = \rho a (V-u)^2$

$$\text{workdone/sec} = F_x \times u$$

$$\text{efficiency, } \eta = \frac{\text{work done/sec}}{\text{K.E of jet}} = \frac{F_x\, u}{\frac{1}{2}(\rho a v)v^2}$$

2. Inclined flat moving plate

$$F_n = \rho a (v-u)^2 \sin\theta$$

$$F_x = \rho a (v-u)^2 \sin^2\theta$$

$$F_y = \rho a (v-u)^2 \sin\theta \cos\theta$$

3 curved moving plate, jet strikes at centre

$$F_x = \rho a (v-u)^2 [1+\cos\theta]$$

$$F_y = -\rho a (v-u)^2 (\sin\theta)$$

4. unsymmetrical moving curved plate, jet strikes at one end.

$$F_x = \rho a V_{r_1} (V_{w_1} + V_{w_2}) \longrightarrow \text{case ①}$$

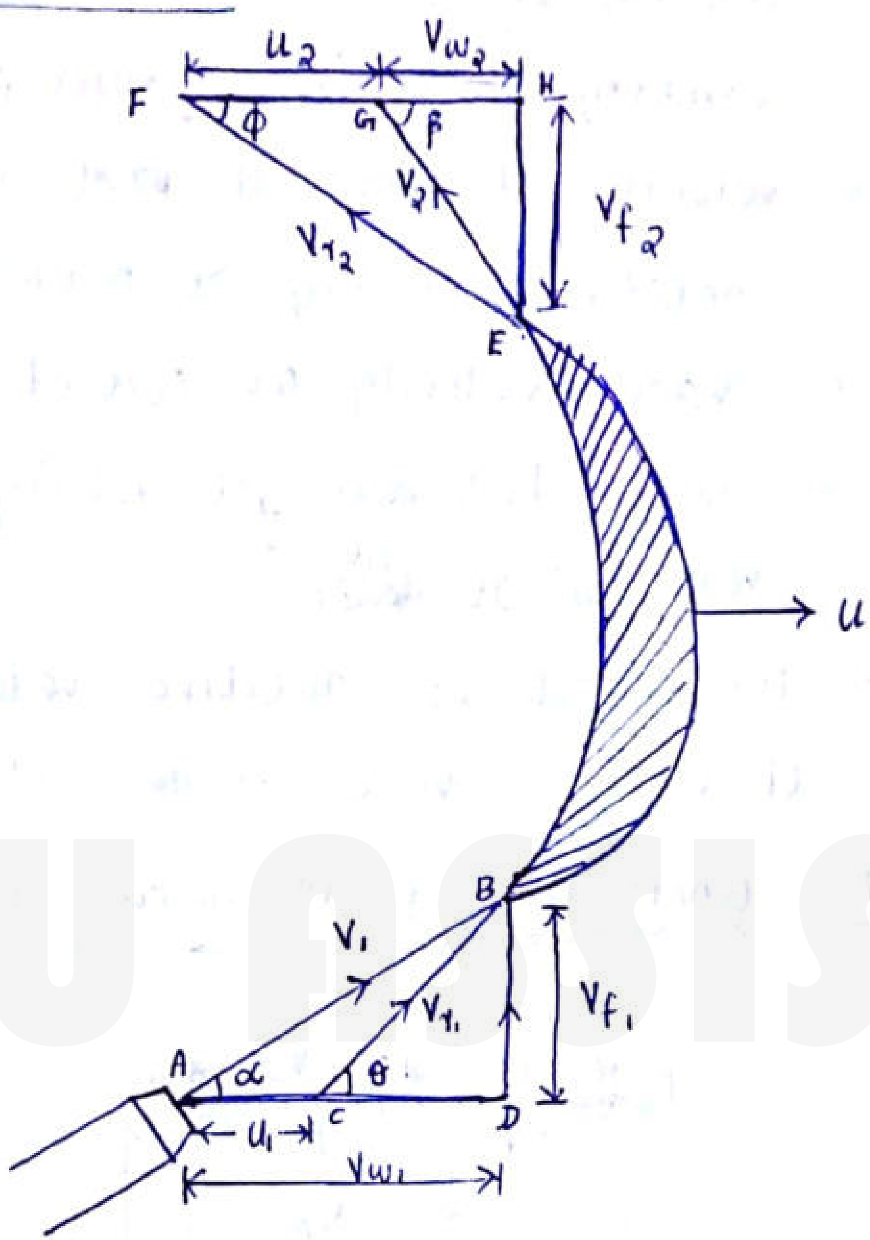
$$F_x = \rho a V_{r_1} (V_{w_1}) \longrightarrow \text{case ②}$$

$$F_x = \rho a V_{r_1} (V_{w_1} - V_{w_2}) \longrightarrow \text{case ③}$$

$$\therefore \boxed{F_x = \rho a V_{r_1} [V_{w_1} \pm V_{w_2}]}$$

08-01-2018

4- Unsymmetrical moving curved plate ; Jet strikes at one of its ends



V_1 = Velocity of jet at inlet

V_{w_1} = velocity of whirl at inlet

V_{f_1} = velocity of flow at inlet

V_{r_1} = Relative velocity at inlet

u_1 - vane velocity at inlet

α - Guide blade angle at inlet

θ = angle between relative velocity and direction of vane = vane angle of inlet

Similiarly.

V_2 = velocity of jet at ~~inlet~~ outlet

V_{w_2} = velocity of whirl at ~~inlet~~ outlet

V_{f_2} = velocity of flow at ~~inlet~~ outlet

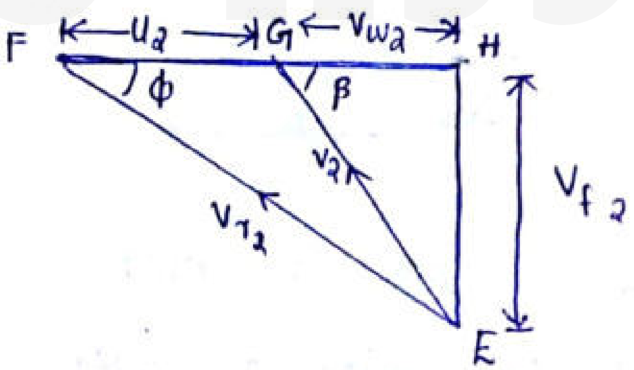
V_{r_2} = relative velocity at outlet

u_2 = vane velocity at outlet

β = angle between jet velocity and vane velocity at outlet

ϕ - Angle between relative velocity and direction of vane = vane angle at outlet

Case - 1 when angle β is acute, ie $\beta < 90^\circ$ at outlet



$V_{r_1} = V_{r_2}$

F_x = mass/sec [inlet velocity - final velocity] in x direction

mass/sec $= \rho a V_{r_1}$

inlet velocity in x-direction $= V_{r_1} \cos\theta = CD = V_{w_1} - u_1$

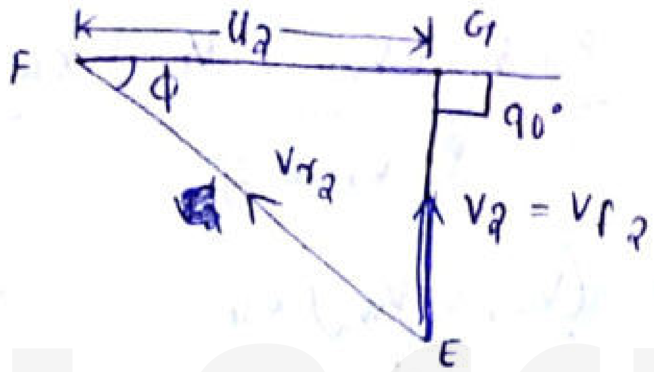
outlet velocity in x-direction $= V_{r_2} \cos\phi = FH = u_2 + V_{w_2}$

$$F_x = \rho a V_{r_1}\left([V_{w_1} - u_1] - -[u_2 + V_{w_2}]\right)$$

$$= \rho a V_{r_1}\left[V_{w_1} - \cancel{u_1} + \cancel{u_2} + V_{w_2}\right]$$

$$\boxed{F_x = \rho a V_{r_1}\left[V_{w_1} + V_{w_2}\right]}$$

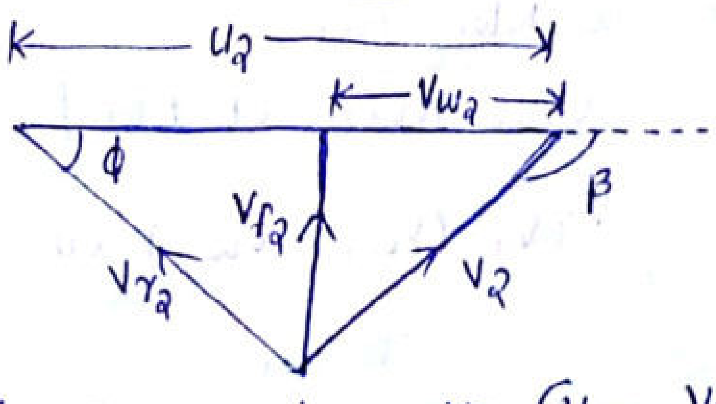
case - 2 when angle $\beta = 90^\circ$



$V_{w_2} = 0$

$$\boxed{F_x = \rho a V_{r_1} V_{w_1}}$$

case - 3 when angle β is obtuse ie, $\beta > 90^\circ$



Final velocity = $V_{r_2} \cos\phi = (u_2 - V_{w_2})$

F_x = mass/sec [inlet velocity - final velocity] in x-direction

$$= \rho a V_{r_1}\left[(V_{w_1} - u_1) - -(u_2 - V_{w_2})\right]$$

$$\boxed{F_x = \rho a V_{r_1}\left[V_{w_1} - V_{w_2}\right]}$$

$$\boxed{\begin{aligned} \text{work done/sec} &= F_x \times u \\ &= \rho a V_{r_1} (V_{w_1} \pm V_{w_2}) \times u \end{aligned}}$$

{unit ⇒ Nm/s}

work done/sec/unit weight of fluid

$$= \frac{\text{work done/sec}}{\text{unit weight of fluid}}$$

$w = mg$

$$= \frac{\cancel{\rho a V_{r_1}} (V_{w_1} \pm V_{w_2}) \times u}{\cancel{\rho a V_{r_1}} \times g}$$

$$= \frac{(V_{w_1} \pm V_{w_2}) \times u}{g}$$

{unit $\Rightarrow \frac{Nm/s}{N/s} = \frac{Nm}{N}$}

work done/sec/unit mass of fluid

$$= \frac{\text{workdone/sec}}{\text{unit mass of fluid}}$$

$$= \frac{\cancel{\rho a V_{r_1}} (V_{w_1} \pm V_{w_2}) \times u}{\cancel{\rho a V_{r_1}}}$$

$$= (V_{w_1} \pm V_{w_2}) \times u$$

{unit ⇒ Nm/kg}

Efficiency of Jet

$$\eta = \frac{\text{output}}{\text{input}}$$

$$= \frac{\text{workdone/sec}}{\text{Initial K.E of jet}}$$

$$\boxed{\eta = \frac{\rho a V_{r_1} (V_{w_1} \pm V_{w_2}) \times u}{\frac{1}{2}(\rho a V_1) \cdot V_1^2}}$$

Q) A jet of water having velocity 40m/s strikes a curved vane, which is moving with a velocity 20m/s. The jet makes an angle of 30° with the direction of motion of a vane at inlet and leaves at an angle of 90° to the direction of motion of vane at outlet. Draw the velocity triangles at inlet and outlet & determine the vane angles at inlet and outlet so that water enters & leaves the vane without shock

given

$V_1 = 40$ m/s

$U = 20$ m/s

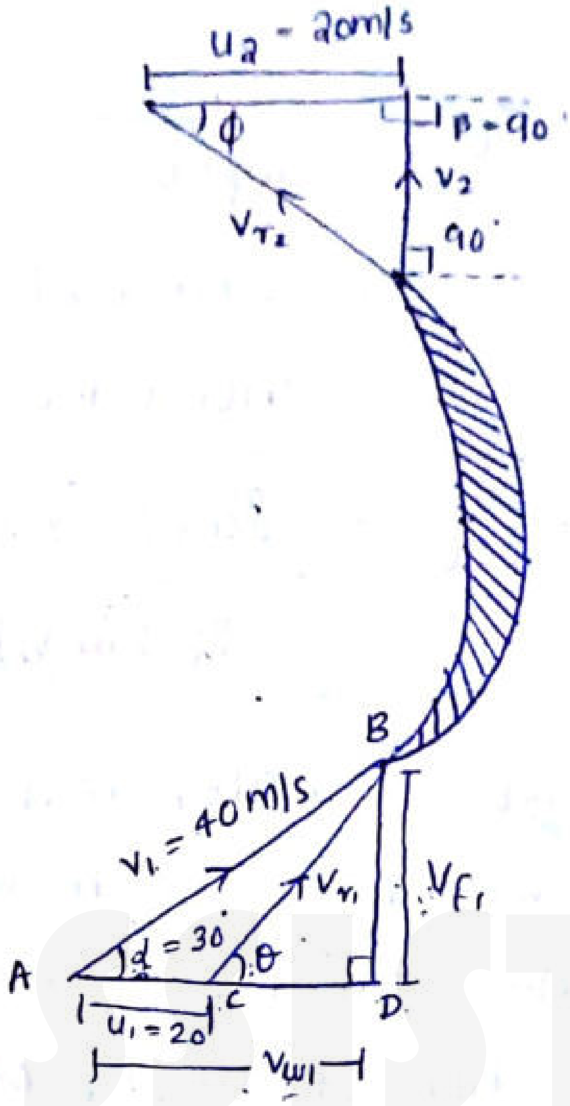
$\alpha = 30°$

$\beta = 180 - 90 = 90°$

$V_{f_1} = V_1 \sin\alpha$

$V_{w_1} = V_1 \cos\alpha$

$\tan\theta = \frac{BD}{CD} = \frac{V_{f_1}}{V_{w_1} - u_1}$



$V_{f_1} = V_1 \sin\alpha$

$= 40 \times \sin 30$

$= \underline{\underline{20 \text{ m/s}}}$

~~Sin θ = Vf1/Vr1~~

$V_{w_1} = V_1 \cos\alpha$

$= 40 \times \cos 30$

$= \underline{\underline{34.641 \text{ m/s}}}$

$\therefore \tan\theta = \frac{V_{f_1}}{V_{w_1} - u_1} = \frac{20}{34.641 - 20}$

$\tan\theta = 1.361$

$\theta = \tan^{-1}(1.361)$

$= \underline{\underline{53.794^\circ}}$

$\sin = \frac{op}{hy}$

$\cos = \frac{ad}{hy}$

$\tan = \frac{op}{ad}$

$\sin 30 = \frac{V_{f_1}}{40}$

$V_f = 40 \cdot$

$\cos 30 = \frac{V_{w_1}}{40}$

$V_{w_1} = 40 \cdot$

$\tan\theta = \frac{V_{f_1}}{V_{w_1}}$

$$\sin\theta = \frac{V_{f1}}{V_{r1}}$$

$$V_{r_1} = \frac{V_{f1}}{\sin\theta} = \frac{20}{\sin 53.794} = \underline{\underline{24.786 \text{ m/s}}}$$

$$\cos\phi = \frac{U_2}{V_{r_2}} = \frac{20}{24.786} = 0.806$$

$$\cos\phi = 0.806$$

$$\phi = \cos^{-1}(0.806)$$

$$= \underline{\underline{36.205^\circ}}$$

Q) A jet of water having a velocity of 20m/s strikes a curved vane which is moving with a velocity of 10 m/s The jet makes an angle of 20° with the direction of motion of vane at inlet and leaves at an angle of 130° to the direction of motion of vane. calculate

(1) vane angles, so that water enters and leaves the vane without shock

(2) work done/sec/unit weight of water

given

$v_1 = 20 m/s$

$U = 10 m/s$

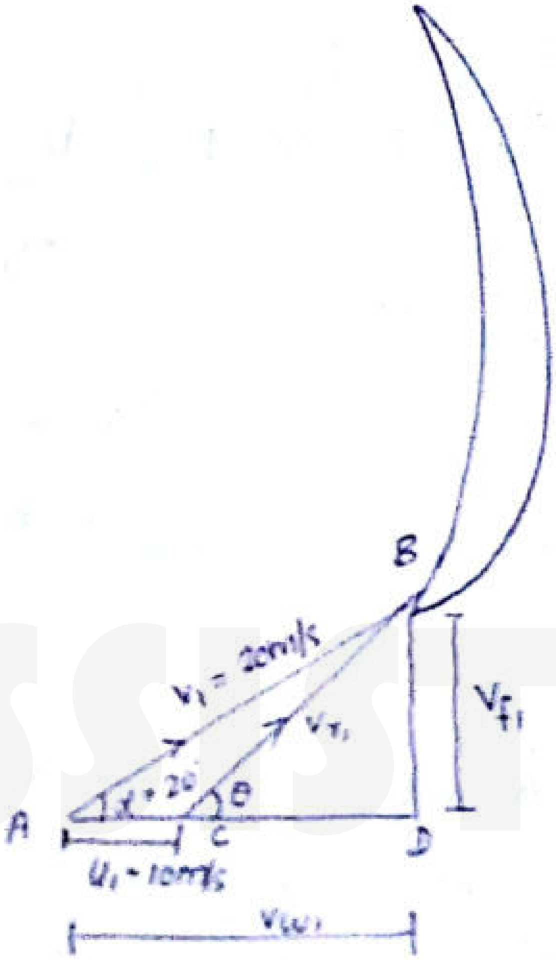
$\alpha = 20^\circ$

$\sin\alpha = \frac{V_{f_1}}{V_1}$

$V_{f_1} = V_1 \sin\alpha$

$= 20 \times \sin 20$

$= 6840$ m/s



$\cos\alpha = \frac{V_{w_1}}{V_1}$

$V_{w_1} = V_1 \cos\alpha$

$= 20 \times \cos 20$

$= 18.793$ m/s

$CD = V_{w_1} - u_1 = 18.793 - 10 = 8.793$

$\tan\theta = \frac{V_{f_1}}{CD} = \frac{6.840}{8.793}$

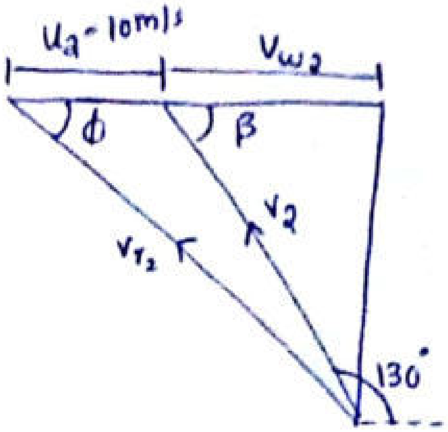
$\theta = \tan^{-1}(0.77)$

$= 37.879^\circ$

$$\sin\theta = \frac{V_{f_1}}{V_{r_1}}$$

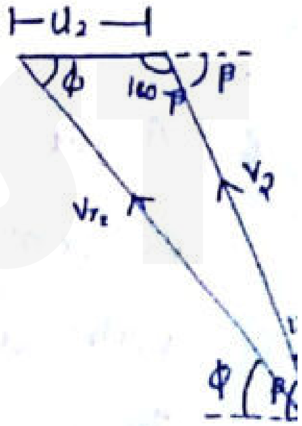
$$V_{r_1} = \frac{V_{f_1}}{\sin\theta} = \frac{6.840}{\sin 37.879} = \underline{\underline{11.140 \text{ m/s}}}$$

By Applying Sine Rule,



$$\frac{V_{r_2}}{\sin(180-\beta)} = \frac{u_2}{\sin(\beta-\phi)} = \frac{V_2}{\sin\phi}$$

$$\frac{11.14}{\sin 130} = \frac{10}{\sin(50-\phi)}$$



$$11.14 \sin(50-\phi) = 10 \sin 130$$

$$\sin(50-\phi) = \frac{7.66}{11.14}$$

$$50-\phi = \sin^{-1}(0.68)$$

$$\phi = 50 - 43.44$$

$$= \underline{\underline{6.55^\circ}}$$

$$\cos\phi = \frac{u_2 + V_{w_2}}{V_{r_2}}$$

$$\cos 6.55 = \frac{10 + V_{w_2}}{11.14}$$

$$11.06 = 10 + V_{w_2}$$

$$V_{w_2} = \underline{\underline{1.06 \text{ m/s}}}$$

$$\text{Workdone/sec/unit weight} = \frac{(V_{w_1} + V_{w_2}) \times u}{g}$$

$$= \frac{(18.793 + 1.06) \times 10}{9.81}$$

$$= \underline{\underline{20.23 \text{ Nm/N}}}$$

Q) A jet of water having diameter 5mm, having a velocity of 20m/s strikes a curved plate which is moving with a velocity of 10m/s in the direction of jet. The jet leaves the vane at an angle of 60° to the direction of motion of vane to the outlet determine

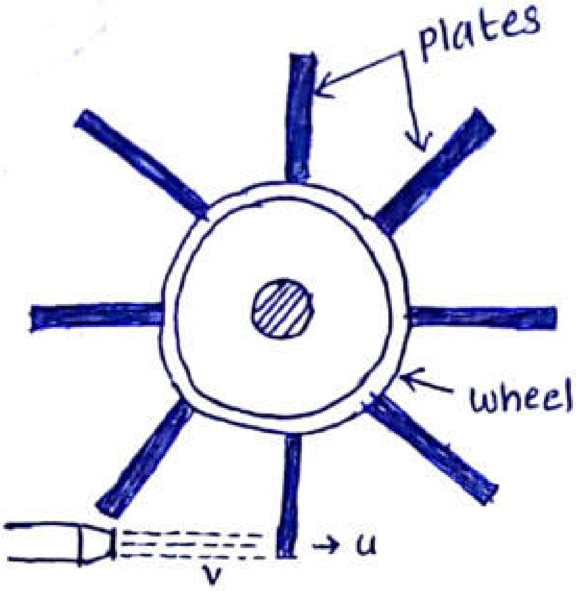
(1) Force excerted by the jet on the vane in the direction of motion

(2) Workdone/sec by the jet

17-01-2018

Force exerted by a jet of water on a series of flat vanes

~~Mas~~



Mass/sec striking on plate $= \rho a v$

Relative velocity $= v - u$

Initial velocity in x-direction $= v - u$

Final velocity in x-direction $= 0$

$\therefore F_x =$ mass/sec (initial velocity − final velocity)

$$= \rho a v \left[(v-u) - 0\right] = \rho a v (v-u)$$

Work done $= F_x \times u = \rho a v (v-u) \times u$

$$\text{Efficiency} = \frac{\text{work done/sec}}{\text{K.E of jet}} = \frac{\rho a v (v-u) u}{\frac{1}{2} (\rho a v) v^2}$$

$$= \frac{2u(v-u)}{v^2}$$

Condition for maximum efficiency

$$\frac{d\eta}{du} = 0$$

$$\frac{d}{du}\left[\frac{2u(v-u)}{v^2}\right] = 0$$

$$\frac{d}{du}\left[\frac{2vu - 2u^2}{v^2}\right] = 0$$

$$\frac{2v - 2 \cdot 2u}{v^2} = 0$$

$$2v - 4u = 0$$

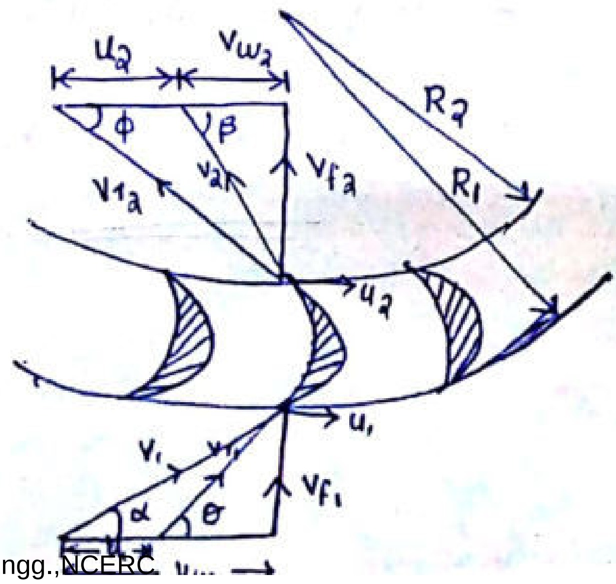
$$2v = 4u$$

$$v = \frac{4}{2}u = 2u$$

$$\boxed{v = 2u}$$

$$\text{Max. efficiency} = \frac{2u(2u-u)}{(2u)^2} = \frac{2u \times u}{2u \times 2u} = \frac{1}{2} = 0.5 = 50\%$$

Force exerted by a series of radial curved vanes



$u_1 = \overset{omega}{\omega} R_1$, $u_2 = \overset{omega}{\omega} R_2$ $\{ u_1 \neq u_2$

$\text{Mass/sec} = \rho a v_1$

Momentum of water at inlet = mass/sec × velocity in x direction

$= \rho a v_1 \times V_{w_1}$

Momentum of water at outlet = mass/sec × velocity at outlet in x-direction

$= \rho a v_1 \times -V_{w_2}$

(-ve sign means opposite direction)

Angular momentum at inlet = momentum × radius at inlet

$= \rho a v_1 \cdot V_{w_1} \cdot R_1$

Angular momentum at outlet $= \rho a v_1 \cdot -V_{w_2} \cdot R_2$

Torque exerted = Rate of change of momentum

= initial momentum − Final momentum

$= \rho a v_1 \, V_{w_1} \, R_1 - (-\rho a v_1 \cdot V_{w_2} \cdot R_2)$

$= \rho a v_1 \left(V_{w_1} R_1 + V_{w_2} R_2 \right)$

$$\text{work done/sec} = \text{Torque exerted} \times \overset{\text{omega}}{\omega}$$

$$= \rho a v_1 (V_{w_1} R_1 + V_{w_2} R_2) \cdot \omega$$

$$= \rho a v_1 (V_{w_1} R_1 \omega + V_{w_2} R_2 \omega)$$

$$= \rho a v_1 \underline{\underline{(V_{w_1} u_1 + V_{w_2} u_2)}} \quad \text{if } \beta \text{ is acute}$$

:t If $\beta = 90^\circ \Rightarrow V_{w_2} = 0$

$$\text{work done/sec} = \rho a v_1 (V_{w_1} u_1)$$

If β is obtuse

$$\text{workdone/sec} = \rho a v_1 (V_{w_1} u_1 - V_{w_2} u_2)$$

Finally

$$\boxed{\text{workdone/sec} = \rho a v_1 (V_{w_1} u_1 \pm V_{w_2} u_2)}$$

$$\omega = \frac{2\pi N}{60}$$

$$\text{Efficiency } \eta = \frac{\text{work done/sec}}{\text{K.E of jet}}$$

$$= \frac{\rho a v_1 [V_{w_1} u_1 \pm V_{w_2} u_2]}{\frac{1}{2}(\rho a v_1) v_1^2}$$

$$\boxed{\eta = \frac{2 [V_{w_1} u_1 \pm V_{w_2} u_2]}{v_1^2}}$$

Flat series vanes

$F_x = \rho a v (v-u)$

work done/sec $= F_x \times u$

$$\eta = \frac{2u(v-u)}{v^2}$$

condition for max. efficiency $\frac{d\eta}{du} = 0$, ie, $v = 2u$

max. efficiency $= \frac{1}{2} = 0.5 = 50\%$

Radial series vanes

work done/sec $= \rho a v_1 (V_{w_1} u_1 \pm V_{w_2} u_2)$

$u_1 = R_1 \omega$ and $u_2 = R_2 \omega$

$$\omega = \frac{2\pi N}{60}$$

$$\eta = \frac{2[V_{w_1} u_1 \pm V_{w_2} u_2]}{v_1^2}$$

1801-2018
Q) A jet of water having a velocity of 30 m/s strikes a series of radial curved vanes mounted on a wheel which is rotating at 200 rpm. The jet makes an angle of 20° with the tangent to the wheel at inlet and leaves the wheel with a velocity of 5 m/s at an angle of 130° to the tangent to the wheel at outlet. Water is flowing from outward in a radial direction. The outer and inner radii of the wheel are 0.5 m and 0.25 m respectively. Determine

(1) vane angle at inlet & outlet

(2) Workdone /sec

(3) work done / unit weight of water

(4) efficiency of the wheel

given

$$V_1 = 30 \text{ m/s}$$

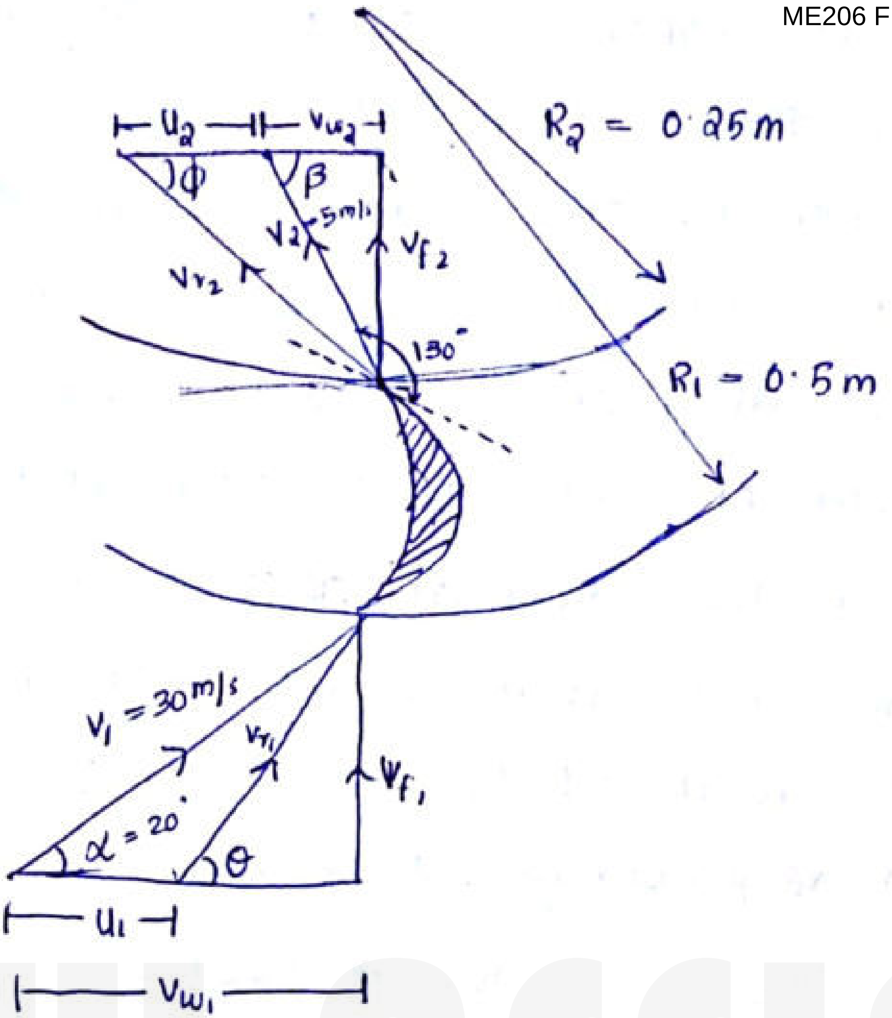
$$N = 200 \text{ rpm}$$

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 200}{60} = \underline{\underline{20.94}}$$

$$\alpha = 20°$$

$$V_2 = 5 \text{ m/s} \quad ; \quad \beta = 180 - 130 = 50°$$

$$R_1 = 0.5 \text{ m}, \quad R_2 = 0.25 \text{ m}$$



$$u_1 = R_1 \omega = 0.5 \times 20.94 = 10.471$$

$$u_2 = R_2 \omega = 0.25 \times 20.94 = 5.235$$

Inlet velocity triangle

$$\sin \alpha = \frac{V_{f1}}{V_1}$$

$$V_{f1} = V_1 \sin \alpha = 30 \times \sin 20 = 10.26 \text{ m/s}$$

$$\cos \alpha = \frac{V_{w1}}{V_1}$$

$$V_{w1} = V_1 \cos \alpha = 30 \times \cos 20 = 28.190 \text{ m/s}$$

$$\tan\theta = \frac{V_{f_1}}{V_{w_1} - u_1} = \frac{10.26}{28.190 - 10.471} = \underline{\underline{0.579}}$$

$$\theta = \tan^{-1}(0.579) = \underline{\underline{30.07^\circ}}$$

outlet velocity triangle

$$\sin\beta = \frac{V_{f_2}}{V_2}$$

$$V_{f_2} = V_2 \sin\beta = 5 \times \sin 50 = \underline{\underline{3.83}}\ m/s$$

$$\cos\beta = \frac{V_{w_2}}{V_2}$$

$$V_{w_2} = V_2 \cos\beta = 5 \times \cos 50 = \underline{\underline{3.21}}\ m/s$$

$$\tan\phi = \frac{V_{f_2}}{V_{w_2} + u_2} = \frac{3.83}{3.21 + 5.235} = 0.453$$

$$\phi = \tan^{-1}(0.453)$$

$$= \underline{\underline{24.39}}$$

work done/sec/unit weight of water $= \dfrac{\cancel{\rho a V_1}\,[V_{w_1} u_1 + V_{w_2} u_2]}{\cancel{\rho a V_1}\cdot g}$

$$= \frac{V_{w_1} u_1 + V_{w_2} \cdot u_2}{g}$$

$$= \frac{28.190 \times 10.471 + 3.21 \times 5.235}{9.81}$$

$$= \underline{\underline{31.802}}$$

$$\eta = \frac{\rho a v_1 [V_{w_1} u_1 + V_{w_2} u_2]}{\frac{1}{2} \rho a v_1 \; v_1^2} = \frac{2(V_{w_1} u_1 + V_{w_2} u_2)}{v_1^2}$$

$$= \frac{2(28.190 \times 10.471 + 3.21 \times 5.235)}{30^2}$$

$$= 0.6932 = \underline{\underline{69.32\%}}$$

Hydraulic machines (turbines)

(1) Gross head

The difference between the tail ~~raise~~ race level and head race level when no water is flowing is known as gross head

(2) Net head

It is the head available at the inlet of turbine when water is flowing through penstock pipe a loss of head due to friction between water and penstock pipe occurs. If h_f is the head loss due to friction between penstock pipe &

water then head, H

$$\boxed{H = H_g - h_f}$$

where $h_f = \dfrac{4 f l v^2}{2 g d}$

19-01-2018

Efficiencies of turbine

1. Hydraulic efficiency (η_h)
2. Mechanical efficiency (η_m)
3. volumetric efficiency (η_v)
4. overall efficiency (η_o)

overall efficiency

Hydraulic efficiency — Mechanical efficiency

Nozzle inlet —— Runner —— Shaft

Hydraulic losses — Mechanical losses.

Overall losses

$\rightarrow \eta_h = \dfrac{R \cdot P}{W \cdot P} = \dfrac{\text{Power delivered to the runner}}{\text{Power supplied at inlet}}$

$\Rightarrow \eta_m = \dfrac{S \cdot P}{R \cdot P} = \dfrac{\text{Power available at shaft}}{\text{Power supplied by the runner}}$

$$\Rightarrow \eta_o = \frac{S.P}{W.P} = \frac{\text{Power available at shaft}}{\text{Power supplied at inlet}}$$

$$= \frac{S.P}{W.P} \times \frac{R.P}{R.P}$$

$$= \frac{S.P}{R.P} \times \frac{R.P}{W.P}$$

$$= \eta_m \times \eta_h$$

water power (w.p) $= \frac{\rho g Q H}{1000}$ or $\frac{W.H}{1000}$ in kW

$$\Rightarrow \eta_v = \frac{\text{volume of water actually striking on the runner}}{\text{Volume of water supplied to the turbine}}$$

classification of turbines.

1. According to type of energy at inlet
   a. Impulse turbine eg:
   b. Reaction turbine
2. According to direction of flow of fluid
   a. Tangential flow eg: Pelton wheel
   b. Radial flow eg: Francis
   c. Axial flow eg: kaplan

d. Mixed flow eg. Modern francis turbine

3 According to type of head available at inlet

a. High head turbine eg: Pelton wheel

b. Medium head turbine eg: Francis

c. Low head turbine eg: kaplan.

4. According to the specific speed of turbine

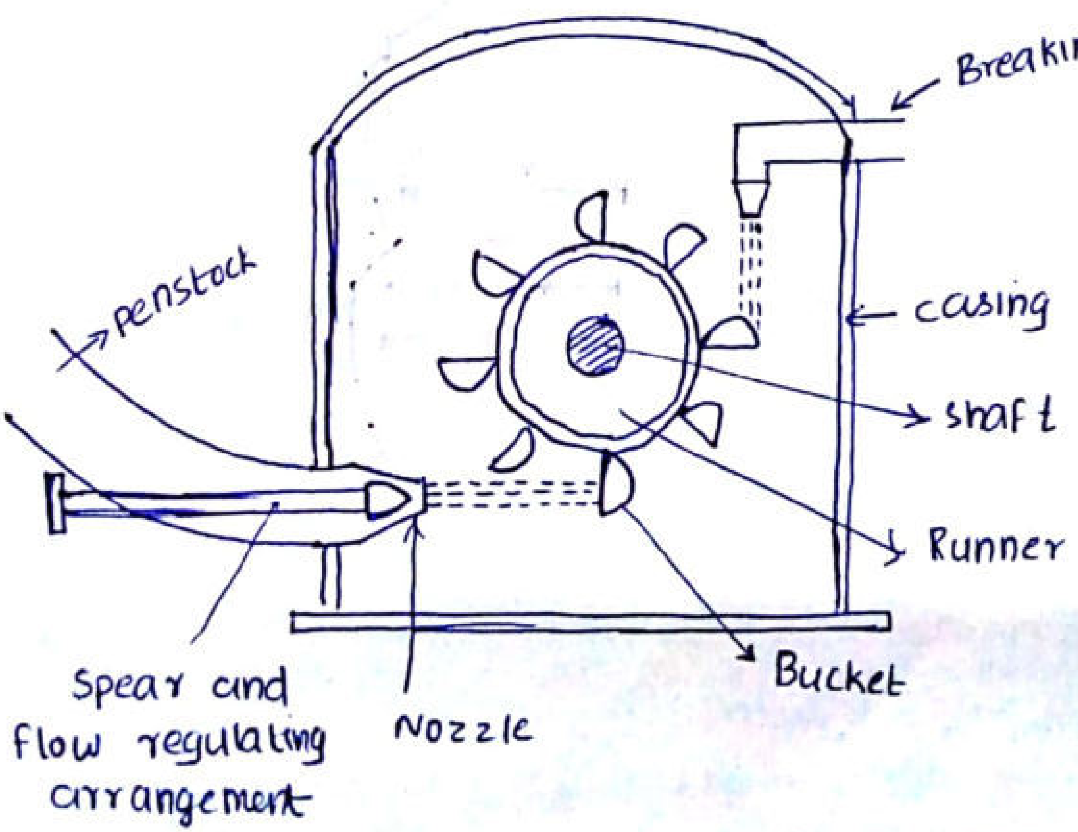
a. High specific speed turbine eg: kaplan.

b. Medium specific speed turbine eg. Francis

c. Low specific speed turbine eg: pelton wheel

22-01-2018

Pelton wheel

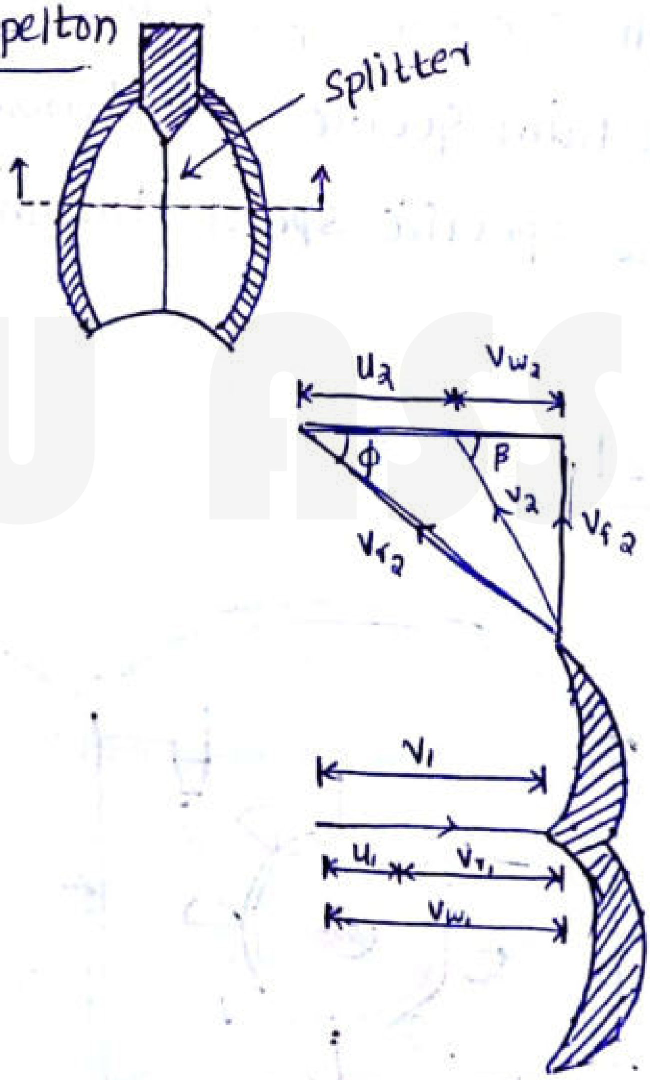


Main 4 parts of pelton wheel

1. Nozzle and flow regulating arrangement
2. Runner and Buckets
3. casing
4. Breaking jet

Velocity triangles of pelton wheel

Bucket of pelton wheel



$$u_1 = u_2 = u = \frac{\pi D N}{60}$$

$$V_1 = C_v \sqrt{2gH}$$

H = net head $= H_g - h_f$

$$h_f = \frac{4fLV^2}{2gD^*}$$

L = Length of penstock

V = velocity in penstock

D^* = Diameter of penstock

D = Dia of Runner

d = dia of jet

Inlet velocity triangle

straight line

$V_{w1} = V_1$

$V_{r1} = V_1 - u_1$

Outlet velocity triangle

$V_{r1} = V_{r2}$

$V_{w2} = V_{r2} \cos\phi - u_2$

$$F_x = \rho a v_1 (V_{w_1} + V_{w_2})$$

$$\text{work done/sec} = F_x \times u$$

$$\text{Power, } P = \frac{F_x \times u}{1000}$$

$$= \frac{\rho a v_1 (V_{w_1} + V_{w_2}) \times u}{1000} \text{ kW}$$

Efficiency

$$\eta_h = \frac{R \cdot P}{W \cdot P}$$

$$= \frac{\rho a v_1 (V_{w_1} + V_{w_2}) \times u}{\frac{1}{2} (\rho a v_1) v_1^2}$$

$$\boxed{\eta_h = \frac{2 (V_{w_1} + V_{w_2}) \times u}{v_1^2}}$$

condition for max. efficiency

$$\frac{d}{du}(\eta_h) = 0$$

$$V_{w_1} = V_1$$

$$V_{w_2} = V_{r_2} \cos\phi - u_2$$

$$= V_{r_1} \cos\phi - u$$

$$= (v_1 - u)\cos\phi - u$$

$$\frac{d}{du}(\eta_h) = 0$$

$$\frac{d}{du}\left[\frac{2\left[v_1 + [(v_1 - u)\cos\phi - u]\right] \times u}{v_1^2}\right] = 0$$

$$\frac{d}{du}\, 2u\left[\frac{(v_1 - u) + (v_1 - u)\cos\phi}{v_1^2}\right] = 0$$

$$\frac{d}{du}\, 2u\left[\frac{(v_1 - u)\,(1 + \cos\phi)}{v_1^2}\right] = 0$$

$$\frac{(1 + \cos\phi)}{v_1^2} \cdot \frac{d}{du}\left[2u\,(v_1 - u)\right] = 0$$

$$\frac{d}{du}\left[2u\,(v_1 - u)\right] = 0$$

$$\frac{d}{du}\left[2uv_1 - 2u^2\right] = 0$$

$$2v_1 - 4u = 0$$

$$2v_1 = 4u$$

$$v_1 = \frac{4u}{2} = \underline{\underline{2u}} \quad \text{or} \quad u = \underline{\underline{\frac{v_1}{2}}}$$

Maximum efficiency

condition ; $u = \frac{V_1}{2}$

$$\eta_h = \frac{2(V_1 - u)(1+\cos\phi)\times u}{V_1^2}$$

$$= \frac{2\left[V_1 - \frac{V_1}{2}\right](1+\cos\phi)\times\frac{V_1}{2}}{V_1^2}$$

$$= \frac{2\times\frac{V_1}{2}\left[1+\cos\phi\right]\times\frac{V_1}{2}}{V_1^2}$$

$$\boxed{\eta_h = \frac{1+\cos\phi}{2}}$$

Design of pelton wheel

① jet velocity , $V_1 = C_v\sqrt{2gH}$

$C_v = 0.98$ or 0.99

H = net head.

② Speed ratio $\phi = \frac{u}{\sqrt{2gH}}$

③ $u = \frac{\pi D N}{60}$ or $D = \frac{60u}{\pi N}$

where $D \rightarrow$ Diameter of runner

④ Jet ratio, $m = \frac{D}{d}$

$d \rightarrow$ diameter of jet

⑤ width of bucket = $5d$

⑥ Depth of bucket = $1.2d$

⑦ No. of bucket on the wheel, $z = 15 + \frac{D}{2d}$

⑧ No. of jets = $\frac{\text{Total discharge}}{\text{Discharge through single jet}}$

Q) A Pelton wheel is to be design for the following specification.

Shaft power (S.P) = 11772 kW

head, H = 380 m

Speed, N = 750 rpm.

Overall efficiency = 86 %

jet diameter, is not to exceed one sixth of the wheel diameter

Determine

(1) The wheel diameter

(2) The no. of jets required.

(3) Diameter of the jet

given, $k_{v_1} = C_v = 0.985$

$k_{u_1} = \phi = 0.45$

$\frac{d}{D} = \frac{1}{6}$

(1) Diameter of wheel

$$V_1 = C_v \sqrt{2gH}$$

$$= 0.985 \times \sqrt{2 \times 9.81 \times 380}$$

$$= \underline{\underline{85.050}}$$

$$\phi = \frac{u}{\sqrt{2gH}}$$

$$u = \phi \cdot \sqrt{2gH}$$

$$= 0.45 \times \sqrt{2 \times 9.81 \times 380}$$

$$= \underline{\underline{38.85}}$$

$$D = \frac{60u}{\pi N} = \frac{60 \times 38.85}{\pi \times 750} = \underline{\underline{0.989\ m}}$$

③ diameter of jet

$$\frac{d}{D} = \frac{1}{6}$$

$$d = \frac{1}{6} \cdot D = \frac{1}{6} \times 0.989 = \underline{\underline{0.164\ m}}$$

$$q = \left(\frac{\pi}{4} d^2\right) \times V_1$$

$$= \frac{\pi}{4} \times 0.164^2 \times 85.050 = \underline{\underline{1.796}}$$

$$\eta_0 = \frac{S.P}{W.P} = \frac{S.P}{\frac{\rho g Q H}{1000}}$$

$$1000\ \eta_0 = \frac{S.P}{\rho g Q . H}$$

$$Q = \frac{S.P \times 1000}{\cancel{1000}\ \eta_0 \cdot \rho g H}$$

$$= \frac{11772 \times 1000}{\cancel{1000 \times} \frac{86}{100} \times 1000 \times 9.81 \times 380} = \underline{\underline{3.67}}$$

$$\therefore \text{No. of jets} = \frac{Q}{q} = \frac{3.67}{1.796} = 2.04 // = \underline{\underline{2\ \text{jets}}}$$

Q) The penstock supplies water from a reservoir to the pelton wheel with a gross head of 500m $1/3^{rd}$ of the gross head is ~~les~~ lost in friction in the penstock, the rate of flow of water through the nozzle fitted at the end of penstock is $2 m^3/s$. The angle of deflection of the jet is 165°. Determine the power given by the water to the runner and also hydraulic efficiency. Take speed ratio $= 0.45$ and $C_v = 1$

given

$H_g = 500m$

$h_f = \frac{H_g}{3} = \frac{500}{3} = 166.66$

$H = H_g - h_f$

$= 500 - 166.66 = 333.33 m$

$Q = 2 m^3/s$

Angle of deflection $= 165°$

Speed ratio, $\phi = 0.45$

$C_v = 1$

Power, $P = \frac{\rho a V_1 (V_{w_1} + V_{w_2}) \times u}{1000}$ kW

$$= \frac{\rho Q \,(V_{w_1} + V_{w_2}) \times u}{1000}$$

$$V_{w_1} = V_1 = C_v \sqrt{2gH}$$

$$= 1 \times \sqrt{2 \times 9.81 \times 333.33}$$

$$= \underline{\underline{80.869}}$$

Speed ratio, $\phi = \dfrac{u}{\sqrt{2gH}}$

$$u = \phi \sqrt{2gH}$$

$$= 0.45 \times \sqrt{2 \times 9.81 \times 333.33}$$

$$= \underline{\underline{36.391}}$$

$$V_{r_1} = V_{r_2} = V_1 - u$$

$$= 80.869 - 36.391 \quad = \underline{\underline{44.478}}$$

$$\cancel{V_{w_2} = V_{r_2} \cos\phi - u}$$

$$\cancel{= (44.478 \times \cos 0.45) - 36.391}$$

$$\cancel{= 8.085}$$

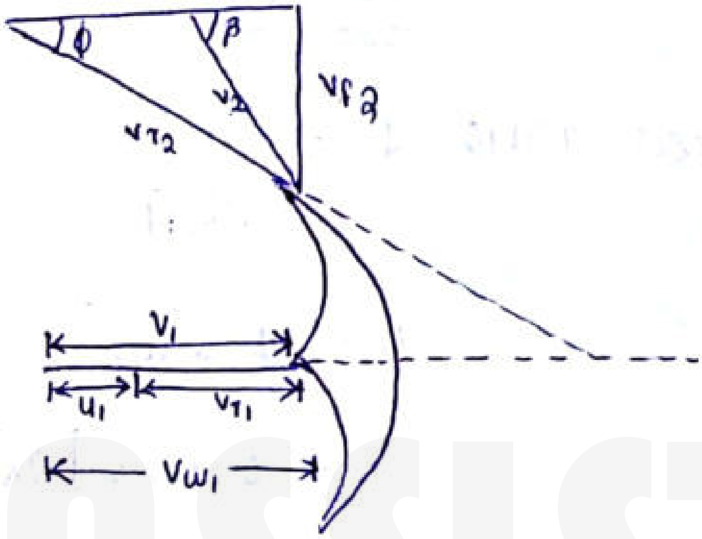
$$\phi = 180 - \text{angle of deflection}$$

$$= 180 - 165 = \underline{\underline{15^\circ}}$$

$$V_{w_2} = V_{r_2}\cos\phi - u$$

$$= 44.478\cos 15 - 36.391$$

$$= \underline{\underline{6.571}}$$



$$\text{Power, } P = \frac{\rho Q\,(V_{w_1} + V_{w_2}) \times u}{1000}$$

$$= \frac{1000 \times 2 \times (80.869 + 6.571) \times 36.391}{1000}$$

$$= \underline{\underline{6363.427}}$$

$$\eta_h = \frac{2\,(V_{w_1} + V_{w_2}) \times u}{V_1^2}$$

$$= \frac{2\,(80.869 + 6.571)\,36.391}{80.869^2} = 0.973 = \underline{\underline{97.3\%}}$$

Q) A pelton wheel is to be designed for a head of 60m when running at 200rpm The pelton wheel develops 95.6475 kW shaft power The velocity of buckets is 0.45 times the velocity of jet, overall efficiency = 0.85 and coefficient of velocity = 0.98

given

$H = 60m$.

$N = 200 rpm$

$S.P = 95.6475 \times 10^3$ W

$\eta_o = 0.85$

$C_v = 0.98$

$V_1 = C_v \sqrt{2gH} = 0.98 \times \sqrt{2 \times 9.81 \times 60} = 33.62$ m/s

velocity of bucket is 0.45 times the velocity of jet

$u_1 = 0.45 V_1$

$u = 0.45 \times 33.62 = 15.130$

Diameter of wheel, $D = \frac{60u}{\pi N} = \frac{60 \times 15.130}{\pi \times 200}$

$= 1.444$ m.

dhs

$$\eta_0 = \frac{S.P}{W.P} = \frac{S.P}{\frac{\rho g Q H}{1000}}$$

$\{Q = a \cdot v_1$
$= \frac{\pi}{4} d^2 \times 33.62$

$$\eta_0 = \frac{1000 \times 95.6475 \times 10^3}{1000 \times 9.81 \times \frac{\pi}{4} \times d^2 \times 33.62 \times 60}$$

$$d^2 = \frac{1000 \times 95.6475 \times 10^3}{1000 \times 9.81 \times \frac{\pi}{4} \times 0.85 \times 33.62 \times 60}$$

$$d = 0.085 \text{ m}$$

No. of bucket on the wheel, $= 15 + \frac{D}{2d}$

$$= 15 + \frac{1.444}{2 \times 0.085}$$

$$= \underline{\underline{23.494}}$$

$$Q = a v_1$$

$$= \frac{\pi}{4} d^2 \times V_1$$

$$= \frac{\pi}{4} \times 0.085^2 \times 33.62 = \underline{\underline{0.190 \text{ m}^3/\text{s}}}$$

Q) The three jet pelton turbine is required to generate 1000 KW under a net head of 400m. The blade angle at outlet is 15° and the reduction in the relative velocity while passing over the blade is 5%. If the overall efficiency of the wheel is 80%, $C_v = 0.98$ and speed ratio = 0.46. Find.

(1) The diameter of the jet

(2) Total flow

(3) Force excerted by a jet on the bucket

given

No. of jets = 3.

S.P (generated power) = 1000 kW.

H (net head) = 400m.

$C_v = 0.98$

Speed ratio, $\phi = 0.46$

$V_{r_2} = 0.95\, V_{r_1}$ {due to reduction of 5%. ie 100 − 5 = 95% = 0.95

$\eta_o = 80\%$

$$\eta_o = \frac{S \cdot P}{\frac{\rho g Q H}{1000}}$$

$$\rho g Q H \eta_o = 1000\, S P$$

$$Q = \frac{1000\ S \cdot P}{\rho g H \cdot \eta_o}$$

$$= \frac{\cancel{1000} \times 10\cancel{00}}{\cancel{1000} \times 9.81 \times 4\cancel{00} \times \frac{80}{100}} = \underline{\underline{0.318\ m^3/s}}$$

Discharge through one jet $= \frac{Q}{3}$

$$= \frac{0.318}{3} = \underline{\underline{0.106\ m^3/s.}}$$

$$V_1 = C_v \sqrt{2gH}.$$

$$= 0.98 \times \sqrt{2 \times 9.81 \times 400}$$

$$= \underline{\underline{86.817\ m/s}}$$

$$\eta_o = \frac{S \cdot P}{\frac{\rho g Q H}{1000}}$$

$$a \cdot V_1\, \rho g\, H \eta_o = 1000\, S P$$

$$\frac{\pi}{4} d^2 = \frac{1000\ \cancel{SP} \times 1000}{86.817 \times 1000 \times 9.81 \times 400 \times 80/100}$$

$$\frac{\pi}{4} d^2 = 3.669 \times 10^{-3}$$

$$d^2 = 4.671 \times 10^{-3}$$

$$d = \underline{\underline{0.068 \text{ m}}}$$

01-02-2018

3. Force excerted by a single jet

$$= \rho Q [V_{w_1} + V_{w_2}]$$

$$V_{w_1} = V_1 = 86.817 \text{ m/s}$$

$$V_{w_2} = V_{r_2} \cos\phi - u_2$$

Speed ratio, $K_u = \dfrac{u_1}{\sqrt{2gH}}$

$$u_1 = K_u \cdot \sqrt{2gH}$$

$$= 0.46 \times \sqrt{2 \times 9.81 \times 400} = \underline{\underline{40.750}}$$

$$u_1 = u_2 = 40.750 \text{ m/s}$$

$$V_{r_1} = V_1 - u_1$$

$$= 86.817 - 40.750$$

$$= \underline{\underline{46.067}}$$

$$V_{r_2} = 0.95 V_{r_1} = 0.95 \times 46.067 = \underline{\underline{43.763}}$$

$$V_{w_2} = V_{r_2} \cos\phi - u_2$$

$$43.763 \cos 15 - 40.750 = 1.521 \text{ m/s}$$

$$F = \rho Q \left[V_{w_1} + V_{w_2} \right]$$

$$= 1000 \times 0.106 \left[86.817 + 1.521 \right]$$

$$= 9363.828 \text{ N}$$

Reaction turbines

- Radial flow turbines
  eg: Francis turbines
  - Inward radial flow turbines
  - outward radial flow turbines
- Axial flow
  eg: Kaplan turbine

Radial flow turbine or Francis turbine

Main parts:-

1. Scroll casing on spiral casing
2. Guide mechanism with guide vanes
3. Runner and runner vanes
4. Draft tubes.

work done /sec $= \rho Q \, (V_{w_1} u_1 \pm V_{w_2} u_2)$

Runner power

$$u_1 = \frac{\pi D_1 N}{60}, \quad u_2 = \frac{\pi D_2 N}{60}$$

12-02-2018

Design aspects of Francis turbine

Force exerted, $F_x = \rho Q \, (V_{w_1} u_1 \pm V_{w_2} u_2)$

V_{w_1} = velocity of whirl at inlet

V_{w_2} = velocity of whirl at outlet

u_1 = tangential velocity at inlet

$= \frac{\pi D_1 N}{60} = R_1 \omega$, ω, angular velocity $= \frac{2\pi N}{60}$

D_1 = Dia of runner at inlet tip

N = angular speed of wheel.

u_2 = tangential velocity at outlet $= \frac{\pi D_2 N}{60} = R_2 \omega$

D_2 = Dia of runner at outlet tip

work done/sec/unit weight $= \frac{\cancel{\rho Q} (V_{w_1} u_1 \pm V_{w_2} u_2)}{\cancel{\rho Q} \times g}$

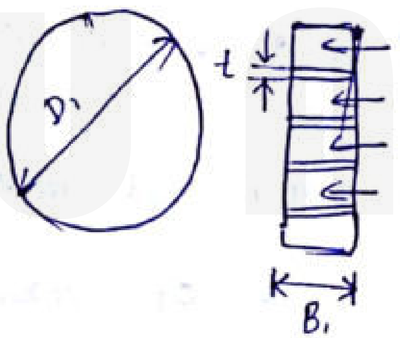
$$= \frac{V_{w_1} u_1 \pm V_{w_2} u_2}{g} \quad Nm/N$$

Speed ratio, $K_u = \frac{u_1}{\sqrt{2gH}}$, K_u varies from 0.6-0.9

Flow ratio, $K_f = \frac{V_{f1}}{\sqrt{2gH}}$, K_f varies from 0.15-0.3

Discharge of the turbine $= \underbrace{\underbrace{\pi D_1}_{\text{perimeter}} \overset{\text{thickness}}{B_1}}_{\text{area}} \times \underbrace{V_{f_1}}_{\text{velocity}} = \pi D_2 B_2 \times V_{f_2}$ $\{Q = a_1 v_1$

$= [\pi D_1 - (n \times t)] B_1 \times V_{f_1}$



B_1 = width of runner at inlet

V_{f_1} = velocity of flow at inlet

n = no. of vanes on the wheel

t = thickness of vane

4. Head on the turbine

$$H = \frac{P_1}{\rho g} + \frac{V_1^2}{2g}$$

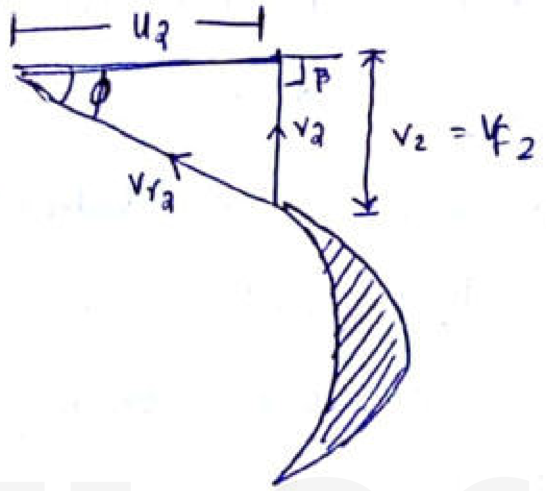
 P_1 = Pressure at inlet

$$H = \frac{(V_{w_1} u_1 + V_{w_2} u_2)}{g} + \frac{V_2^2}{2g}$$

03-02-2018

Special practical cases

1. Discharge is radial at outlet

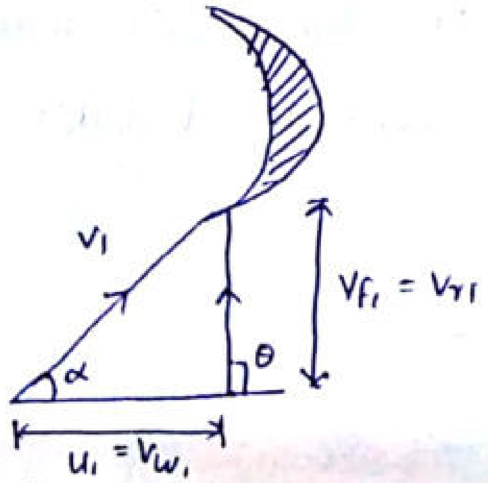


$V_2 = V_{f_2}$

$V_{w_2} = 0$

$\angle \beta = 90^\circ$

2. Runner vane are radial at inlet or radial inlet



$u_1 = V_{w_1}$

$V_{r_1} = V_{f_1}$

$\angle \theta = 90^\circ$

Hydraulic efficiency

$$\eta_h = \frac{R \cdot P}{W \cdot P} = \frac{\rho Q (V_{w_1} u_1 \pm V_{w_2} u_2)/1000}{\rho g Q H/1000}$$

{Converting into kW dividing by 1000 on numerator & deno

$$\eta_h = \frac{V_{w_1} u_1 \pm V_{w_2} u_2}{gH}$$

If outlet is radial ; $V_{w_2} = 0$

$$\eta_h = \frac{V_{w_1} u_1}{gH}$$

Q) A Francis turbine with an over all efficiency of 75% is required to produce 148.25 kW power. It is working under a head of 7.62 m. The peripheral velocity is $0.26\sqrt{2gH}$ and radial velocity of flow at inlet is $0.96\sqrt{2gH}$. The wheels ~~runner~~ runs at 150 rpm and hydraulic losses in the turbine are 22% of the available energy. Assuming radial discharge. Find

(1) Guide blade angle

(2) The vane angle at inlet

(3) diameter of wheel at inlet

(4) width of wheel at inlet

given

$\eta_o = 75\%$

$SP = 148.25$ kW

$H = 7.62$ m.

peripheral velocity, $u_1 = 0.26\sqrt{2gH}$

velocity of flow, $V_{f_1} = 0.96\sqrt{2gH}$

Speed = 150 rpm.

Hydraulic losses = 22%

Discharge at outlet = radial

$$\therefore u_1 = 0.26\sqrt{2\times 9.81\times 7.62} = 3.179$$

$$V_{f_1} = 0.96\sqrt{2\times 9.81\times 7.62} = 11.738$$

Speed ratio, $k_u = \frac{u_1}{\sqrt{2gH}} \Rightarrow u_1 = k_u\sqrt{2gH}$

Flow ratio, $k_f = \frac{V_{f_1}}{\sqrt{2gH}} \Rightarrow V_{f_1} = k_f\sqrt{2gH}$

$$\therefore k_u = 0.26$$

$$k_f = 0.96$$

Hydraulic losses = 22%

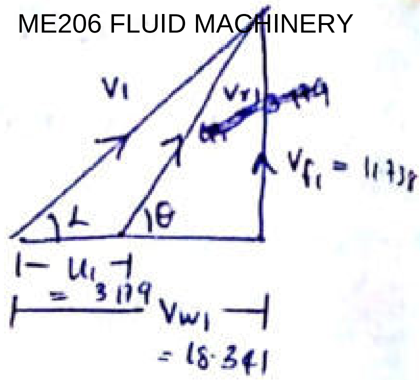
Hydraulic efficiency = $100 - 22 = 78\%$

$$\frac{V_{w_1} u_1}{gH} = \frac{78}{100}$$

$$V_{w_1} = \frac{78}{100} \times \frac{gH}{u_1}$$

$$= \frac{78}{100} \times \frac{9.81\times 7.62}{3.179}$$

$$= 18.341$$



$$\tan\alpha = \frac{V_{f_1}}{V_{w_1}}$$

$$\alpha = \tan^{-1}\left[\frac{11.738}{18.341}\right]$$

$$= \underline{\underline{32.618^\circ}}$$

$$\tan\theta = \frac{V_{f_1}}{V_{w_1} - u_1}$$

$$\theta = \tan^{-1}\left[\frac{11.738}{18.341 - 3.179}\right]$$

$$= \underline{\underline{37.746^\circ}}$$

$$u_1 = \frac{\pi D_1 N}{60}$$

$$D_1 = \frac{60 u_1}{\pi N} = \frac{60 \times 3.179}{\pi \times 150} = \underline{\underline{0.404 \text{ m}}}$$

$$\eta_0 = \frac{SP}{\frac{\rho g Q H}{1000}}$$

$$\rho g Q H \eta_0 = 1000\, SP$$

$$Q = \frac{1000\, SP}{\rho g H \eta_0} = \frac{1000 \times 148.25}{1000 \times 9.81 \times 7.62 \times \frac{75}{100}} = \underline{\underline{2.644}}$$

$$Q = \pi D_1 B_1 \times V_{f_1}$$

$$B_1 = \frac{Q}{\pi D_1 V_{f_1}} = \frac{2.644}{\pi \times 0.404 \times 11.738} = \underline{0.177\ m}$$

MODULE - 2

Kaplan turbine

Parts

1. Spiral or scroll casing
2. Guide wheel mechanism with guide vanes
3. Runner and runner vanes
4. Draft tube

Design aspects of kaplan turbine

1. Discharge, $Q = \frac{\pi}{4}(D_o^2 - D_b^2) \times V_{f_1}$

D_o = outer diameter of runner

D_b = dia. of hub or boss

2. Tangential velocity of runner

$$u_1 = u_2 = \frac{\pi D_o N}{60}$$

Starting & ending of flow through outlet so D_o

3. velocity of flow at inlet and outlet are equal

$$V_{f_1} = V_{f_2}$$

4. Area of flow at inlet and outlet are equal $= \frac{\pi}{4}(D_o^2 - D_b^2)$

5. Speed ratio, $K_u = \frac{u_1}{\sqrt{2gH}}$

6. Flow ratio, $K_f = \frac{V_{f_1}}{\sqrt{2gH}}$

08-02-18

Q) A kaplan turbine working under head of 20m develops 11772 kW shaft power. The outer dia of runner is 3.5 m and hub dia is 1.75m. The guide blade angle at the extreme edge of runner is 35°. The hydraulic and overall efficiencies of the turbine are 88% and 84% respectively. If the velocity of whirl is zero at outlet determine

(1) Runner vane angles at inlet and outlet at the extreme edge of the runner

(2) Speed of the turbine

given

$H = 20m$

$SP = 11772\ kW$

$D_o = 3.5\ m$

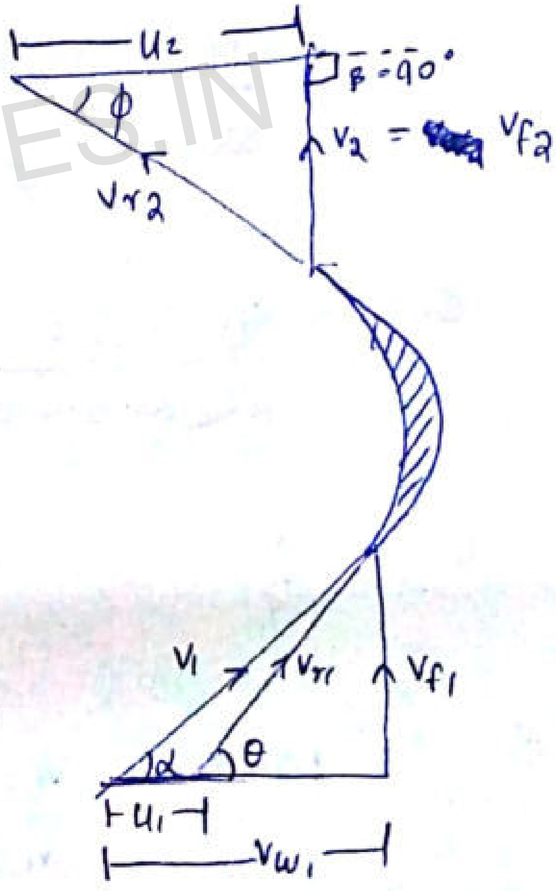
$D_b = 1.75m$.

$\alpha = 35^\circ$

$\eta_h = 88\%$

$\eta_o = 84\%$

$V_{w2} = 0$



$$\eta_o = \frac{SP}{WP}$$

$$= \frac{SP}{\rho g Q H / 1000}$$

$$\eta_o = \frac{SP \times 1000}{\rho g \times \frac{\pi}{4}(D_o^2 - D_b^2) V_{f_1} \times H}$$

$$V_{f_1} = \frac{SP \times 1000}{\rho g \times \frac{\pi}{4}(D_o^2 - D_b^2) \times H}$$

$$= \frac{11772 \times 1000}{1000 \times 9.81 \times \frac{\pi}{4} \times (3.5^2 - 1.75^2) \times 20 \times 0.84}$$

$$= \underline{\underline{9.898 \text{ m/s}}}$$

$$Q = \frac{\pi}{4}(3.5^2 - 1.75^2) \times 9.89 = \underline{\underline{71.364 \text{ m}^3/\text{s}}}$$

12-02-2018

$$\tan\alpha = \frac{V_{f_1}}{V_{w_1}}$$

$$V_{w_1} = \frac{V_{f_1}}{\tan\alpha} = \frac{9.898}{\tan 35} = \underline{\underline{14.135 \text{ m/s}}}$$

$$\eta_H = \frac{V_{w_1} u_1 + V_{w_2} u_2}{gH} \qquad \{ V_{w_2} = 0$$

$$\eta_H = \frac{V_{w_1} u_1}{gH}$$

$$u_1 = \frac{\eta_H \cdot g \cdot H}{V_{w_1}}$$

$$= \frac{\frac{88}{100} \times 9.81 \times 20}{14.135} = \underline{\underline{12.214 \text{ m/s}}}$$

$$\tan\theta = \frac{V_{f_1}}{V_{w_1} - u_1} = \frac{9.898}{14.135 - 12.214} = \underline{\underline{5.152}}$$

$$\therefore \theta = \tan^{-1}(5.152) = \underline{\underline{79.016^\circ}}$$

$$\tan\phi = \frac{V_{f_2}}{u_2} \qquad \left\{ \begin{array}{l} V_{f_1} = V_{f_2} = 9.898 \\ u_1 = u_2 = 12.214 \end{array} \right.$$

$$\phi = \tan^{-1}\left[\frac{9.898}{12.214}\right]$$

$$= \underline{\underline{39.020^\circ}}$$

Speed of turbine

$$u_1 = u_2 = \frac{\pi D_o N}{60}$$

$$N = \frac{60\, u_1}{\pi D_o} = \frac{60 \times 12.214}{\pi \times 3.5} = \underline{66.648 \text{ rpm}}$$

Q) A kaplan turbine develops 24647.6 kW power at an average head of 39 m. Assuming a speed ratio of 2 and flow ratio = 0.6; dia of the boss = 0.35 times the dia of runner and an η_o = 90%. Calculate the diameter, speed and specific speed of the turbine

<u>given</u>

$SP = 24647.6$ kW

$H = 39$ m.

$K_u = 2$

$K_f = 0.6$

$D_b = 0.35\ D_o$

$\eta_o = 90\%$

$$K_u = \frac{u_1}{\sqrt{2gH}} \quad , \quad K_f = \frac{V_{f_1}}{\sqrt{2gH}}$$

① Diameter of turbine (D_0)

$$\eta_0 = \frac{S.P}{\rho g Q H / 1000}$$

$$\eta_0 = \frac{1000 \; SP}{\rho g Q H}$$

$$Q = \frac{1000 \; SP}{\rho g \eta_0 H}$$

$$= \frac{1000 \times 24647.6}{1000 \times 9.81 \times 90/100 \times 39} = \underline{\underline{71.581 \; m^3/s}}$$

$$V_{f_1} = K_f \cdot \sqrt{2gH}$$

$$= 0.6 \times \sqrt{2 \times 9.81 \times 39} = \underline{\underline{16.597 \; m/s}}$$

$$\therefore Q = \frac{\pi}{4} \left(D_0^2 - D_b^2 \right) V_{f_1}$$

$$71.581 = \frac{\pi}{4} \left[D_0^2 - 0.35^2 D_0^2 \right) \times 16.597$$

$$D_0^2 \left[1 - 0.35^2 \right] = \frac{4 \times 71.581}{\pi \times 16.597} = 5.491$$

$$D_0^2 = 6.257 \Rightarrow D_0 = \underline{2.501 \; m}$$

(2) $u_1 = K_u \sqrt{2gH}$

$= 2\sqrt{2 \times 9.81 \times 39}$

$= \underline{\underline{55.323 \text{ m/s}}}$

$N = \frac{60 u_1}{\pi D_0} = \frac{60 \times 55.323}{\pi \times 2.501} = \underline{\underline{422.467 \text{ rpm}}}$

$D_b = 0.35 D_0$

$= 0.35 \times 2.501 = \underline{\underline{0.875 \text{ m}}}$

Specific speed,

$N_s = \frac{N\sqrt{P}}{H^{5/4}} = \frac{422.467 \times \sqrt{24647.6}}{39^{5/4}}$

$= \underline{\underline{680.533 \text{ rpm}}}$

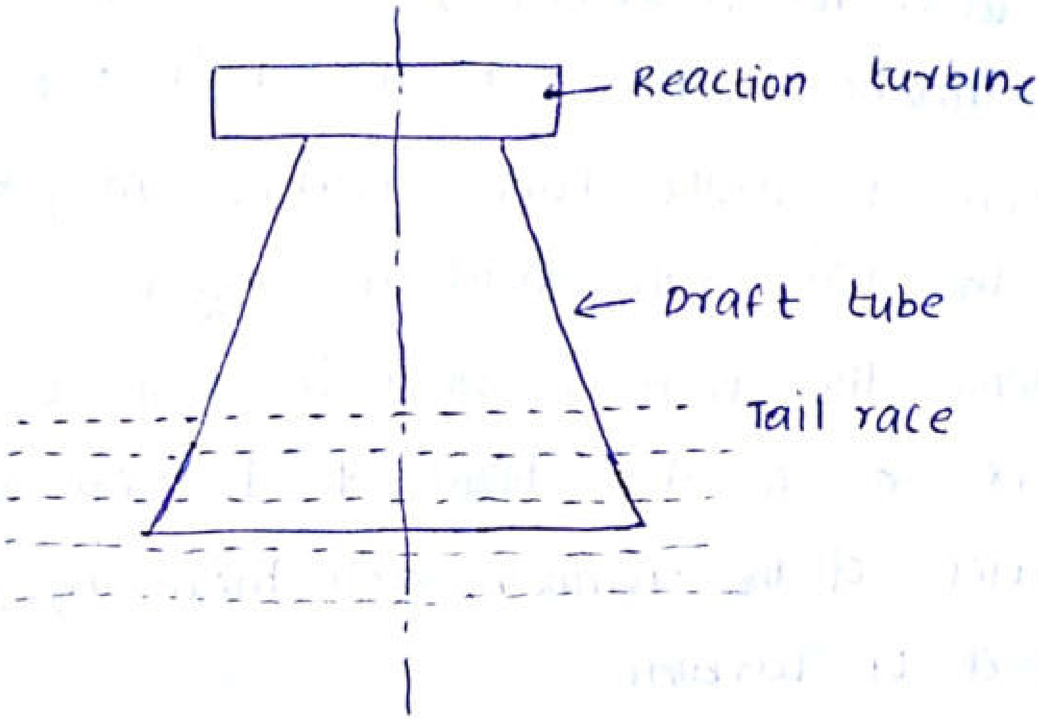
$$\boxed{\text{Specific Speed, } N_s = \frac{N\sqrt{P}}{H^{5/4}}}$$

where, N - Speed of turbine

P → shaft power

H - Head.

Draft tube



- continously increasing cross sectional area
- Kinetic head to pressure head due to increasing area.
- Total pressure head increased.
- turbine ~~head~~ notch can be placed at higher head

Draft tube is a pipe of gradually increasing area which connects the outlet of runner to the tail race. One end of draft tube is connected to the outlet of the runner while the other end is submerged below the level of water in the tail race.

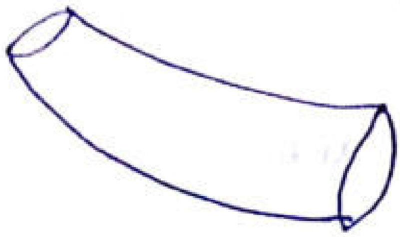
Functions

- It is used for discharging water from the exit of the reaction turbine to the tail race
- with the help of draft tube, turbine may be placed above the tail race without any loss of net head and hence the turbine may be inspected properly
- Permits a negative head to be established at the outlet of the runner and there by increase the net head of turbine
- It converts the large proportion of Kinetic energy rejected at the outlet of turbine in to useful pressure energy. ~~without~~ ~~th~~
- without the draft tube, the K.E rejected at the outlet of the turbine will go waste to the tail race

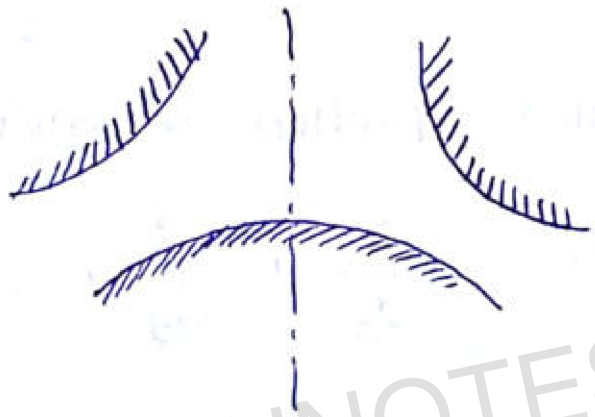
Types of Draft tube.

① Conical draft tube

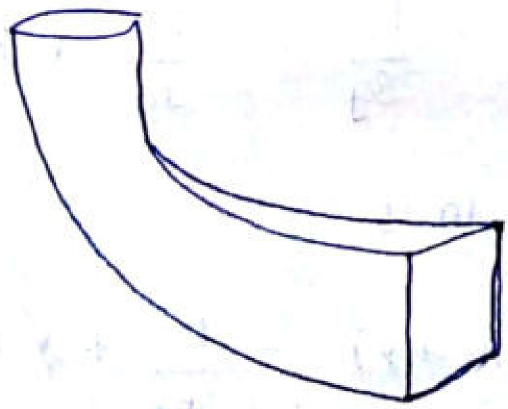
② Elbow draft tube



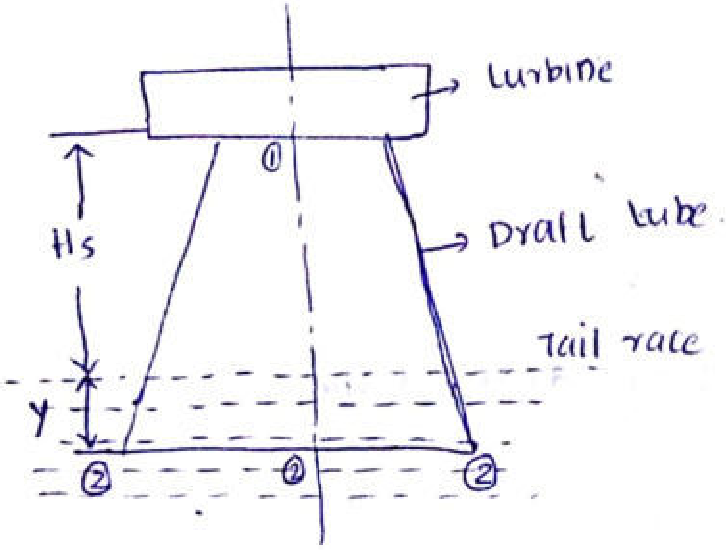
③ Moody spreading draft tube.



④ Draft tube with circular inlet and rectangular outlet



Draft tube theory



taking section ①-② as datum head.

By applying Bernoulli's equation at inlet and outlet of draft tube

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + H_f$$

{head loss due to friction

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + (H_s + y) = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + 0 + h_f \quad \text{—①}$$

Pressure head at outlet, $\frac{P_2}{\rho g} = \frac{P_a}{\rho g} + y$ { P_a = atmospheric pressure

Sub. in ①

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + (H_s + \cancel{y}) = \frac{P_a}{\rho g} + \cancel{y} + \frac{V_2^2}{2g} + h_f$$

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + H_s = \frac{P_a}{\rho g} + \frac{V_2^2}{2g} + h_f$$

$$\frac{P_1}{\rho g} = \frac{P_a}{\rho g} + \frac{V_2^2}{2g} + h_f - \frac{V_1^2}{2g} - H_s$$

$$\frac{P_1}{\rho g} = \frac{P_a}{\rho g} - H_s - \underbrace{\left[\frac{V_1^2}{2g} - \frac{V_2^2}{2g} - h_f\right]}$$

Pressure head at inlet is the difference b/w atmospheric pressure and the 4 different terms so that the pressure head at inlet become negative and the flow towards the draft tube inlet become higher and high power can be extracted from it.

where, H_s = vertical height of draft tube about Tail race

Y = distance of bottom of draft tube from tail race

From the above equation it is clear that the pressure head at section ①-① is less than atmospheric pressure head.

Efficiency of draft tube

It is defined as actual conversion of kinetic head into pressure head in the draft tube to the kinetic head at the inlet of draft tube.

efficiency of draft tube

$$= \frac{\text{actual conversion of kinetic head in to pressure head.}}{\text{Kinetic head at inlet}}$$

$$\boxed{\eta = \frac{\left[\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right] - h_f}{V_1^2/2g}}$$

15-02-18

Specific Speed (N_s)

MP

unit → rpm

$$\text{specific speed } (N_s) = \frac{N\sqrt{P}}{H^{5/4}}$$

It is defined as the speed of a turbine which is identical in shape, geometrical dimensions, blade angles etc with the actual turbine but of such a size that it will develop one kW power when working under unit head

(IMP) Derivation of specific speed of a turbine

overall efficiency, $\eta_o = \frac{S.P}{W.P} = \frac{P}{\frac{\rho g Q H}{1000}}$

$\therefore P = \eta_o \cdot \frac{\rho g Q H}{1000}$

$P \propto QH$ —— ①

Speed ratio, $K_u = \frac{u_1}{\sqrt{2gH}}$

$u_1 = K_u \sqrt{2gH}$

$u_1 \propto \sqrt{H}$ —— ②

$u_1 = \frac{\pi D N}{60}$

$u_1 \propto DN$ —— ③

Comparing ② and ③

$DN \propto \sqrt{H}$

$D \propto \frac{\sqrt{H}}{N}$ —— ④

Velocity, $= C_v \sqrt{2gH}$

$$V \propto \sqrt{H} \quad — (5)$$

Discharge, Q = area x velocity

$$Q \propto D \times B \times \sqrt{H}$$

$B \propto D$.

$$Q \propto D^2 \sqrt{H} \quad — (6)$$

Sub (4) in (6)

$$Q \propto \left[\frac{\sqrt{H}}{N}\right]^2 \sqrt{H}$$

$$Q \propto \frac{H}{N^2} \sqrt{H}$$

$$Q \propto \frac{H^{3/2}}{N^2} \quad — (7)$$

Sub (7) in (1)

$$P \propto \frac{H^{3/2}}{N^2} \times H$$

$$P \propto \frac{H^{5/2}}{N^2}$$

$$P = K \frac{H^{5/2}}{N^2} \quad — (8)$$

For obtaining value of k

By applying principle of specific speed.

$H = 1\,m, \quad P = 1\,kW, \quad N = N_s$

$$1 = \frac{K \cdot 1^{5/2}}{N_s^2}$$

$$N_s^2 = K$$

Sub value of k in eq (8)

$$P = N_s^2 \cdot \frac{H^{5/2}}{N^2}$$

$$N_s^2 = \frac{P \cdot N^2}{H^{5/2}}$$

$$N_s = \frac{N\sqrt{P}}{\sqrt{H^{5/2}}}$$

$$\boxed{N_s = \frac{N\sqrt{P}}{H^{5/4}}}$$

Significance of specific speed

Specific speed place an important role for selecting the type of the turbine also the perfomance of the turbine can be predicted by knowing the specific speed of the turbine

Sl No	Specific speed	Types of turbine
1	8.5 to 30	Pelton wheel with single jet
2	30 to 51	Pelton wheel with 2 or more jets
3	51 to 225	Francis turbine
4	225 to 860	Kaplan turbine or propeller turbine

Unit Quantities

1. Unit speed (N_u)
2. Unit discharge (Q_u)
3. Unit Power (P_u)

In order to predict the behaviour of a turbine working under varying conditions of head, speed, output and gate opening. The results are expressed in terms of quantities which may be obtained when head on the turbine is reduced to unity that means 1m

① Unit Speed (N_u)

It is defined as the speed of the turbine working under a unit head. ie under head of 1 m. It is denoted by N_u

$$H = 1\,m, \quad N = N_u$$

Speed ratio, $K_u = \dfrac{u}{\sqrt{2gH}}$

$$u = K_u \times \sqrt{2gH}$$

$$u \propto \sqrt{H} \quad \text{—①}$$

Tangential velocity, $u = \dfrac{\pi D N}{60}$

$u \propto N$ —② [D is constant for a turbine]

From ① and ②

$$N \propto \sqrt{H}$$

$$N = K\sqrt{H} \quad \text{—③}$$

$$N_u = K\sqrt{1}$$

$$N_u = K$$

Substitute $K = N_u$ in ③

$$N = N_u \sqrt{H}$$

$$\boxed{N_u = \frac{N}{\sqrt{H}}}$$

② <u>Unit discharge</u> (Q_u)

It is defined as the discharge passing through a turbine which working under unit head. ie under a head of 1m

Discharge, Q = Area x velocity

$$V = C_v \sqrt{2gH}$$

$$V \propto \sqrt{H} \quad \text{—①}$$

Flow ratio, $K_{f_1} = \dfrac{V_{f_1}}{\sqrt{2gH}}$

$$V_{f_1} = K_f \times \sqrt{2gH}$$

$$V_{f_1} \propto \sqrt{H}$$

Area of flow is constant for a turbine

$$Q = AV$$

$$Q \propto V \quad \text{only}$$

$$V \propto \sqrt{H}$$

ie, $Q \propto V \propto \sqrt{H}$

From this, $Q \propto \sqrt{H}$

$$Q = K_2 \sqrt{H} \quad \text{—(2)}$$

k_2 = Constant of propotionality

For obtaining the value of k_2

$H = 1m$, $Q = Q_u$ {sub in (2)

$$Q_u = K_2 \sqrt{1}$$

$$K_2 = Q_u$$

Sub the value of k_2 in (2) equation.

$$Q = Q_u \sqrt{H}$$

$$\boxed{Q_u = \frac{Q}{\sqrt{H}}}$$

3 Unit Power (P_u)

It is defined as the power developed by a turbine working under unit head ie, under a head of 1 m.

overall efficiency, $\eta_o = \dfrac{S.P}{\frac{\rho g Q H}{1000}}$

Shaft power, $P = \eta_0 \cdot \frac{\rho g Q H}{1000}$

$P \propto QH$ —①

$Q \propto \sqrt{H}$ —② {From the derivation of unit discharge

$\therefore P \propto \sqrt{H} \cdot H$

$P \propto H^{3/2}$

$P = k_3 H^{3/2}$ —③

$\left\{ H^{1/2} \cdot H = H^{1/2+1} = H^{\frac{1+2}{2}} = H^{3/2} \right.$

where k_3 = Constant of propotionality

For obtaining the value of k_3

$H = 1m$ and $P = P_u$ {Sub in ③

$P_u = k_3 \cdot 1^{3/2}$

$\rightarrow k_3 = P_u$ //

Sub in ③

$P = P_u \cdot H^{3/2}$

$$\boxed{P_u = \frac{P}{H^{3/2}}}$$

Use of Unit Quantities

If a turbine is working under different heads the behaviour of a turbine can be easily known from the values of unit Quantities.

$$N_u = \frac{N_1}{\sqrt{H_1}} = \frac{N_2}{\sqrt{H_2}}$$

$$Q_u = \frac{Q_1}{\sqrt{H_1}} = \frac{Q_2}{\sqrt{H_2}}$$

$$P_u = \frac{P_1}{H_1^{3/2}} = \frac{P_2}{H_2^{3/2}}$$

where, $H_1, H_2 \Rightarrow$ The different heads under which the turbine works

$N_1, N_2 \Rightarrow$ The corresponding speeds

$Q_1, Q_2 \Rightarrow$ The corresponding discharge

$P_1, P_2 \Rightarrow$ Corresponding power developed by the turbine

Q) A turbine is to operate under a head of 25m at 200 rpm. The discharge is 9m³/sec. If the efficiency is 90%, determine the performance of a turbine under a head of 20m.

given,

$H_1 = 25\ m$

$N_1 = 200\ rpm$

$Q_1 = 9\ m^3/sec$

$\eta_0 = \frac{90}{100} = \underline{0.9}$

$H_2 = 20m$

$$\frac{N_1}{\sqrt{H_1}} = \frac{N_2}{\sqrt{H_2}}$$

$$N_2 = \frac{N_1\sqrt{H_2}}{\sqrt{H_1}} = \frac{200 \times \sqrt{20}}{\sqrt{25}} = \underline{178.885}\ rpm$$

$$\frac{Q_1}{\sqrt{H_1}} = \frac{Q_2}{\sqrt{H_2}}$$

$$Q_2 = \frac{Q_1\sqrt{H_2}}{\sqrt{H_1}} = \frac{9 \times \sqrt{20}}{\sqrt{25}} = \underline{8.049\ m^3/sec}$$

$$\eta_o = \frac{P}{\frac{\rho g Q H}{1000}}$$

$$P = \eta_o \frac{\rho g Q H}{1000}$$

$$P_1 = \frac{90}{100} \times \frac{\cancel{1000} \times 9.81 \times 9 \times 25}{\cancel{1000}} = \underline{\underline{1986.525 \text{ kW}}}$$

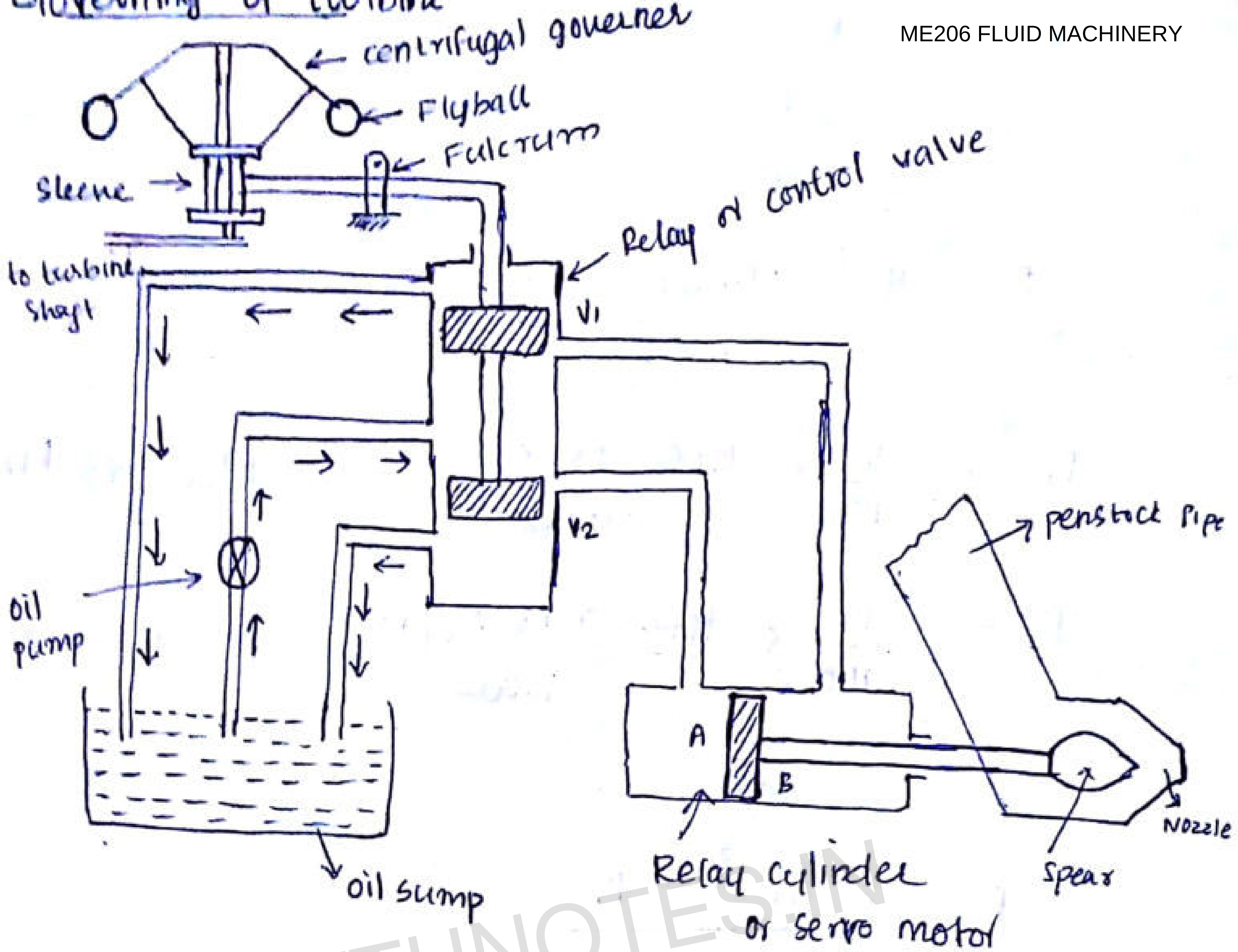
$$P_2 = \frac{90}{100} \times \frac{\cancel{1000} \times 9.81 \times 8.049 \times 20}{\cancel{1000}} = \underline{\underline{1421.292 \text{ kW}}}$$

OR

$$P_2 = \frac{H_2^{3/2} \cdot P_1}{H_1^{3/2}}$$

$$= \frac{20^{3/2} \times 1986.525}{25^{3/2}} = \underline{\underline{1421.441 \text{ kW}}}$$

Governing of turbine



The governing of a turbine is defined as the operation by which the speed of turbine is kept constant under all conditions of working. So this done automatically by means of ~~centrifugal~~ a governer which regulates the rate of flow ~~to~~ through the turbine according to the changing load condition of the turbine.

Governing of pelton turbine is done by means of oil pressure governer which consist of following parts

(1) oil sump
(2) Gear pump also called oil pump which is driven by power obtained from the turbine shaft.
(3) The control valve or relay valve or distribution valve
(4) servo motor also called relay cylinder
(5) The centrifugal governer which is driven by belt or gear from the turbine shaft
(6) The pipe connecting the oil sump to the control valve and control valve with servomotor
(7) spear rod or needle

22-02-2018

Surge tank

Surge tank is a storage reservoir fitted at penstock pipe near to the power house before the valve

Functions

1) when the load on the generator is reduced, turbine spear valve or wicket gates are closed for reducing the rate of flow. These valves due to sudden closing of these valves, large amount of water moving towards the turbine push backwards. The rejected water is then stored in the surge tank. Hence by reducing water hammer

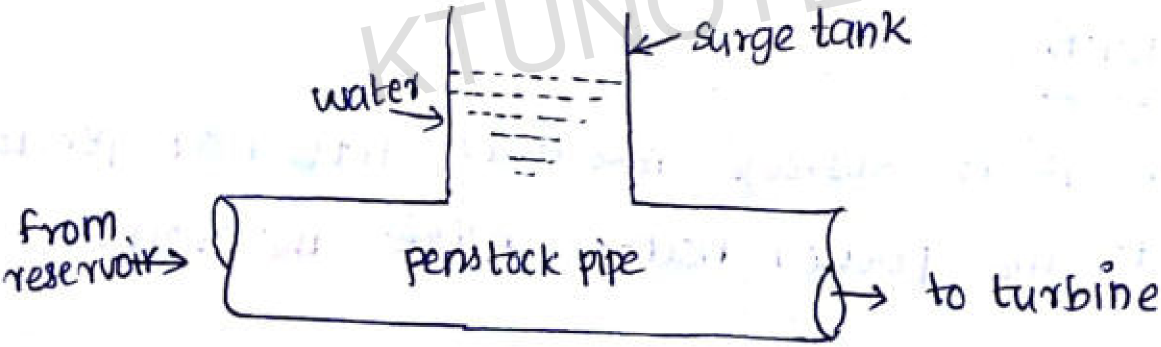
effect on penstock

a) when the load on the generator increases, the governor opens the spear values or wicket gates to increase the rate of flow entering the runner. The increased demand of water by the turbine is partially fulfilled by supplying water from the surge tank

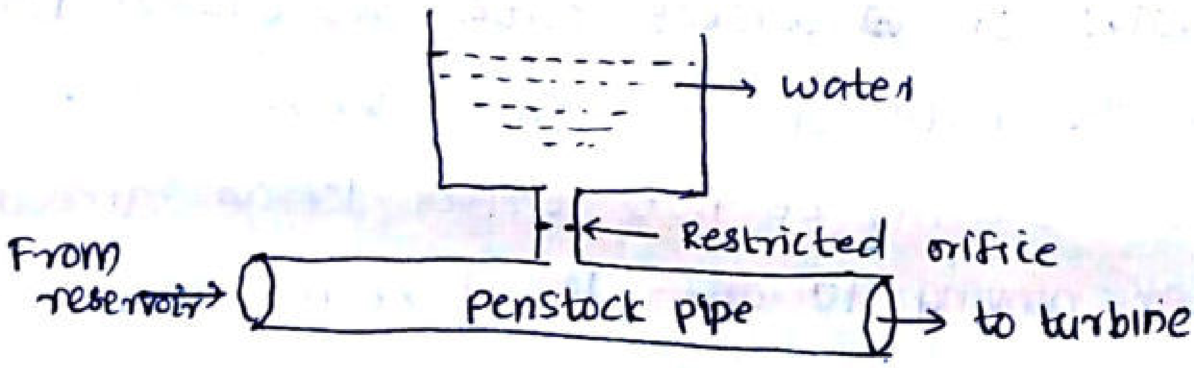
Types of surge tank

① Simple surge tank
② Restricted orifice surge tank
③ Diffrential surge tank

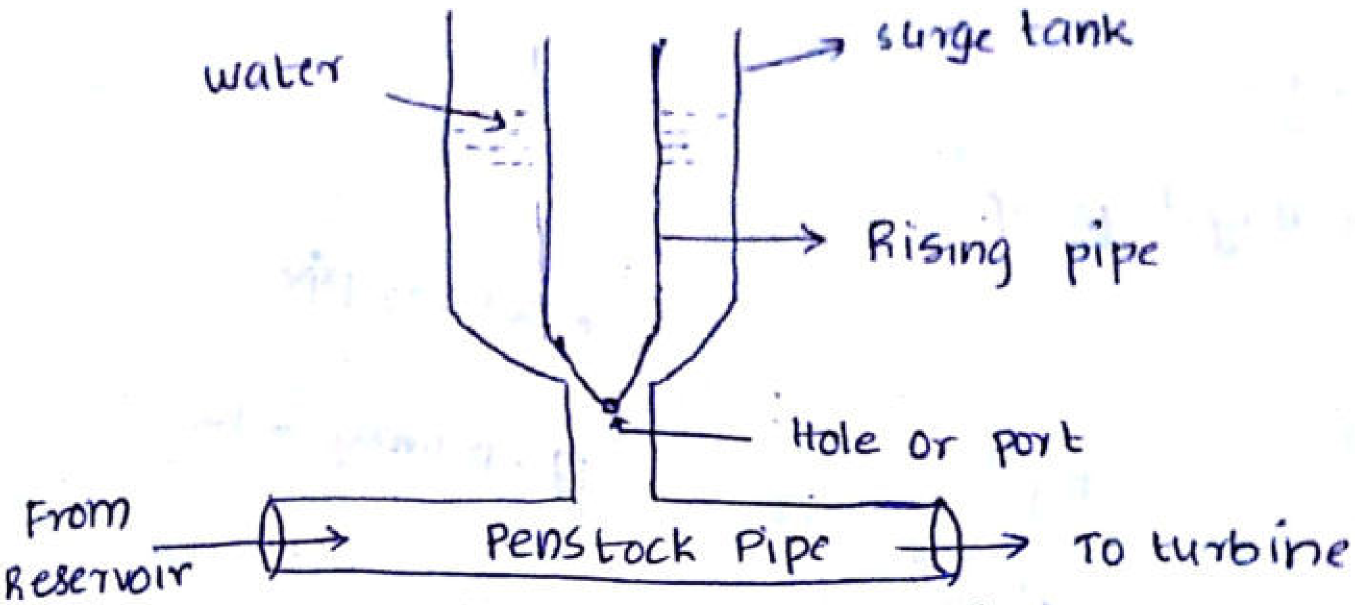
① simple surge tank



② Restricted orifice surge tank



(3) Diffrential Surge tank

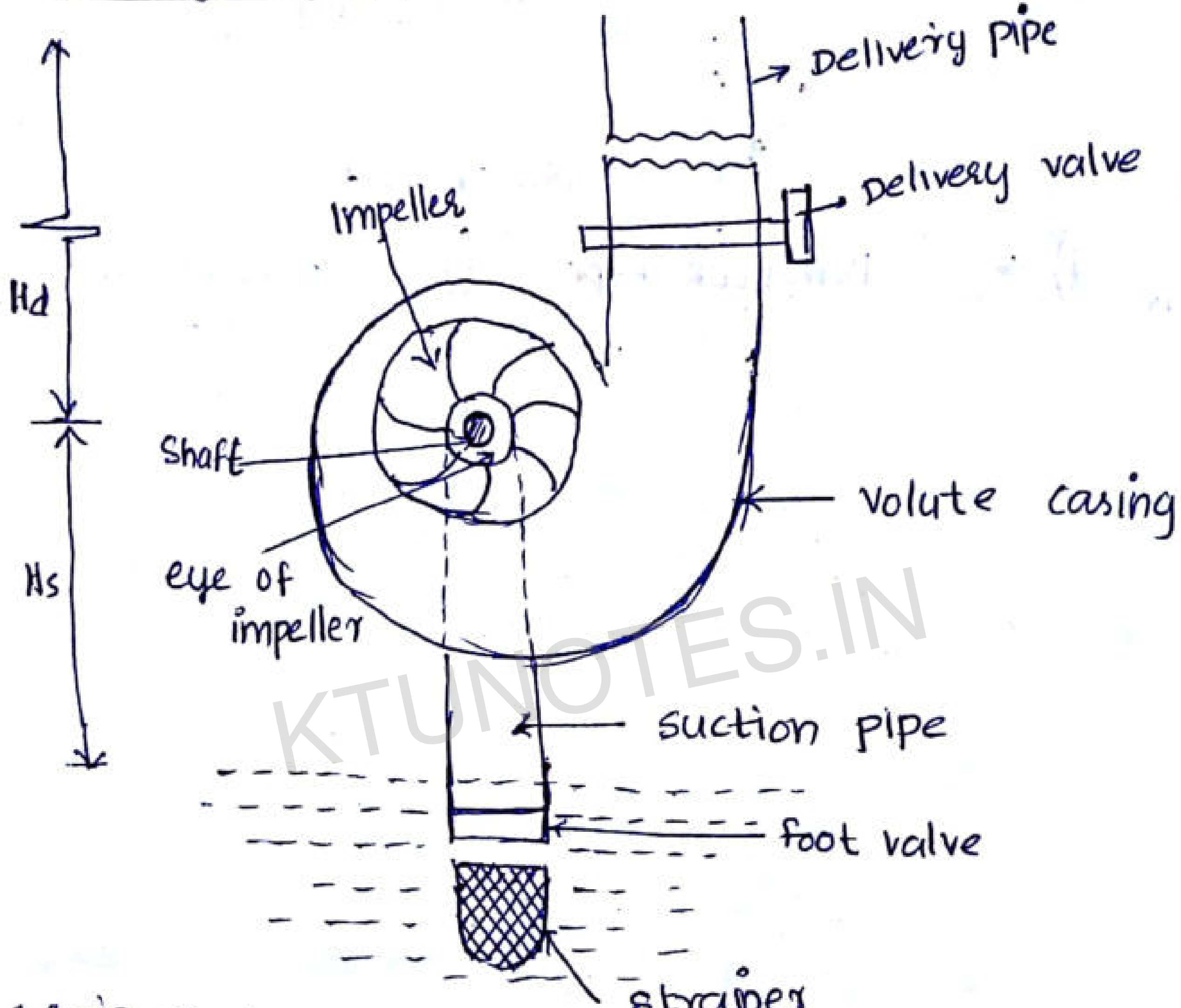


22-02-2018

MODULE -3

Pump

Centrifugal pump

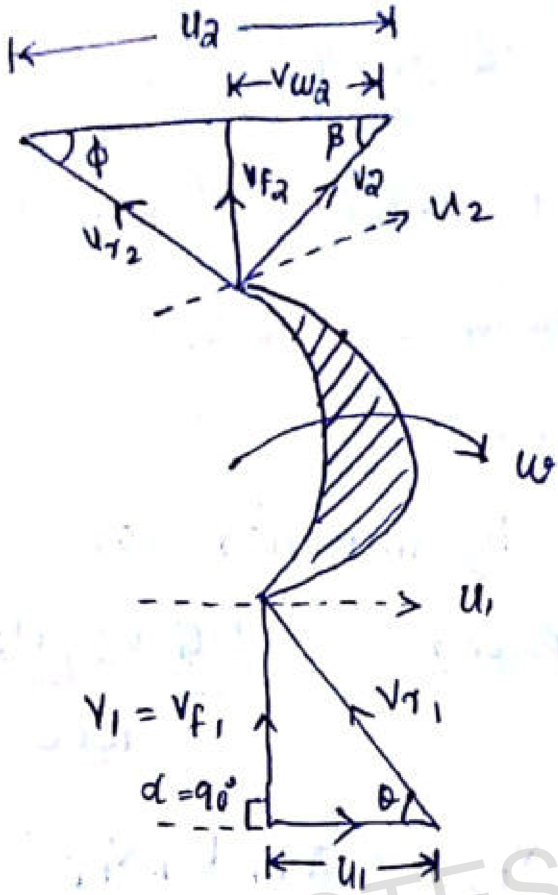


Main parts

1. Impeller
2. case
3. Suction pipe with foot valve and strainer
4. Delivery pipe with regulating valve

21-02-18

Velocity triangle and work done of centrifugal pump.



$V_{w_1} = 0$

$V_1 = V_{f_1}$

ω = Angular velocity $= \frac{2\pi N}{60}$

u_1 = tangential velocity of vane at inlet

$= \frac{\pi D_1 N}{60} = \omega R_1$

D_1 = Diameter of ~~pipe~~ impeller at inlet of vane

u_2 = tangential velocity of vane at outlet

$= \frac{\pi D_2 N}{60} = \omega R_2$

D_2 = Diameter of impeller at outlet of vane

$w = mg$
$= \rho g v$
$= \rho g Q$

Radial discharge at inlet

$\alpha = 90^\circ$, $V_1 = V_{f_1}$, $V_{w_1} = 0$, $V_{f_1} = V_{f_2}$

workdone/sec $= \rho Q V_{w_2} u_2$

workdone/sec/unit weight of water $= \frac{\cancel{\rho Q} V_{w_2} u_2}{\cancel{\rho Q} g}$

β obtuse $-[\rho Q (V_{w_1} u_1 - V_{w_2} u_2)]$
$= \rho Q V_{w_2} u_2$

$= \frac{V_{w_2} u_2}{g}$

Power developed by impeller or

Impeller power $= \frac{\rho Q V_{w_2} u_2}{1000}$ kW

$Q = \pi D_2 B_2 V_{f_2} = \pi D_1 B_1 V_{f_1}$, B = ~~width~~ width

Various heads on centrifugal pump.

1. suction head or suction lift (h_s)
2. Delivery head or delivery lift (h_d)
3. static head (H)
4. Euler's head (H_E)
5. Manometric head (H_m)

① **Suction head (h_s)**

vertical height of the center line of centrifugal pump above the water surface in sump from which water is to be lifted.

② **Delivery head (h_d)**

vertical distance b/w the centre line of pump and water surface in the tank to which water is delivered.

③ **Static head**

$$H = h_s + h_d$$

④ **Eulers head (H_E)**

It's theoretical head imparted by impeller on water. If no energy losses in impeller and casing then,

Impeller power = water power

$$\cancel{\rho}\cancel{Q} V_{w_2} u_2 = \cancel{\rho} g \cancel{Q} H$$

$$V_{w_2} u_2 = gH$$

$$\boxed{H_E = \frac{V_{w_2} u_2}{g}}$$

⑤ Manometric head (H_m)

It is the head against which the centrifugal pump has to work.

(a) H_m = head imparted by the impeller − pump losses

$$= \frac{V_{w_2} u_2}{g} - \text{pump losses}$$

{ pump losses = 0 (If there is no pump loss)

(b) $H_m = h_s + h_d + h_{fs} + h_{fd} + \frac{V_d^2}{2g}$

h_s = Suction head.

h_d = delivery head.

h_{fs} = frictional loss in suction pipe

h_{fd} = Frictional loss in delivery pipe

V_d = velocity in delivery pipe

(c) $H_m = \left[\frac{P_o}{\rho g} + \frac{V_o^2}{2g} + Z_o\right] - \left[\frac{P_i}{\rho g} + \frac{V_i^2}{2g} + Z_i\right]$

$\frac{P_o}{\rho g}$ = pressure head at outlet

$\frac{V_o^2}{2g}$ = velocity head at outlet

Z_o = datum heat at outlet

$\frac{P_1}{\rho g}$, $\frac{V_1^2}{2g}$, Z_1 are the corresponding pressure, velocity, datum head at inlet

Efficiencies of centrifugal Pump.

① Manometric or hydraulic efficiency (η_{mano})

$$\eta_{mano} = \frac{\text{manometric head}}{\text{Eulers or theoretical head}}$$

$$= \frac{H_m}{\frac{V_{w_2} u_2}{g}} = \frac{g H_m}{V_{w_2} u_2}$$

(or)

$$\eta_{mano} = \frac{\text{water power}}{\text{impeller power}}$$

$$= \frac{\rho g Q H_m}{\rho Q V_{w_2} u_2} = \frac{g H_m}{V_{w_2} u_2}$$

② Mechanical efficiency

$$\eta_{mech} = \frac{\text{Impeller power}}{\text{Shaft power}} = \frac{\rho Q V_{w_2} u_2}{\frac{2\pi N T}{60}}$$

③ Overall efficiency

$$\eta_{overall} = \frac{\text{water power}}{\text{shaft power}} = \eta_{mano} \times \eta_{mech}$$

Q) The internal and external diameter of impeller is of centrifugal pump are 200mm and 400mm respectively. Pump run at 1200 rpm the vane angle of impeller at inlet and outlet are 20° and 30° respectively. The water enters impeller radially and velocity of flow is constant. Determine work done by the impeller per unit weight of work.

given

$D_1 = 200mm = 0.2\,m$

$D_2 = 400mm = 0.4\,m.$

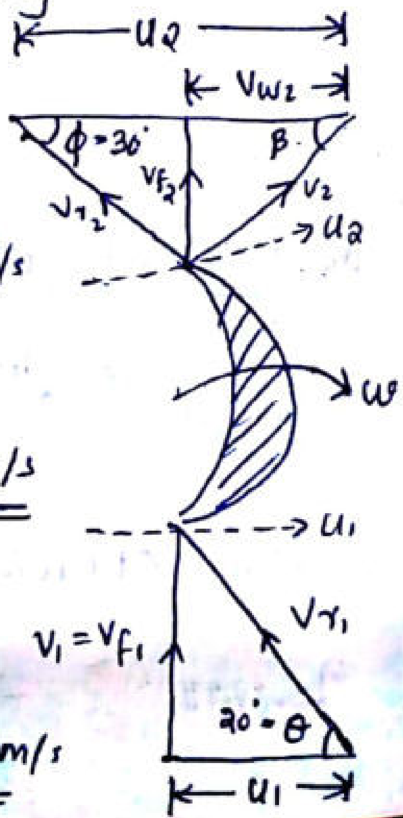
$N = 1200\ rpm.$

$\theta = 20°$

$\phi = 30°$

$V_{w_1} = 0,\ \alpha = 90° \Rightarrow$ radially

$V_{f_1} = V_{f_2} \Rightarrow$ flow constant



$u_1 = \dfrac{\pi D_1 N}{60} = \dfrac{\pi \times 0.2 \times 1200}{60} = \underline{\underline{12.56\ m/s}}$

$u_2 = \dfrac{\pi D_2 N}{60} = \dfrac{\pi \times 0.4 \times 1200}{60} = \underline{\underline{25.13\ m/s}}$

$\tan\theta = \dfrac{V_{f_1}}{u_1}$

$V_{f_1} = 12.56 \times \tan 20° = \underline{\underline{4.573\ m/s}}$

$V = V_{f_1} = V_{f_2} = 4.573\ m/s$

$$\tan\phi = \frac{V_{f_2}}{u_2 - V_{w_2}}$$

$$\tan 30 = \frac{4.573}{25.13 - V_{w_2}}$$

$$25.13 - V_{w_2} = \frac{4.573}{\tan 30}$$

$$V_{w_2} = 25.13 - \frac{4.573}{\tan 30} = \underline{\underline{17.209\ m/s}}$$

$$\text{work done/sec/unit weight} = \frac{V_{w_2} u_2}{g}$$

$$= \frac{17.209 \times 25.13}{9.81}$$

$$= \underline{\underline{44.083\ Nm/N}}$$

Q) A centrifugal pump discharge $0.118\ m^3/s$ at a speed of 1450 rpm against head of 25m the impeller dia is 250mm. its width at outlet is 50mm. manometer efficiency 75% determine vane angle at outer periphery of impeller.

<u>given</u>

$Q = 0.118\ m^3/s$

$N = 1450\ rpm$

Impeller outer dia, $D_2 = 250mm = 0.25\ m$

width, $B_2 = 50mm = 0.05\ m$

$\eta_{mano} = 75\% = 0.75$

$$U_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.25 \times 1450}{60} = \underline{\underline{18.98\ m/s}}$$

$$Q = \pi D_2 B_2 V_{f_2}$$

$$V_{f_2} = \frac{Q}{\pi D_2 B_2} = \frac{0.118}{\pi \times 0.25 \times 0.05} = \underline{\underline{3.004\ m/s}}$$

$$\eta_{mano} = \frac{g H_m}{V_{w_2} \cdot U_2}$$

$$V_{w_2} = \frac{g H_m}{\eta_{mano} \times U_2}$$

$$= \frac{9.81 \times 25}{0.75 \times 18.98} = \underline{\underline{17.22\ m/s}}$$

$$\phi = \tan^{-1}\left[\frac{V_{f_2}}{U_2 - V_{w_2}}\right]$$

$$= \tan^{-1}\left[\frac{3.004}{18.98 - 17.22}\right] = \underline{\underline{59.634^\circ}}$$

28-02-'18

8) A centrifugal pump delivers water against a net head of 14.5 m and a design speed of 1000 rpm. The vanes are curved back at an angle of 30° with the periphery. The impeller (outer) diameter is 300mm and outlet width is 50mm. Determine the discharge of pump if manometric efficiency is 95%.

given

Head, $H = 14.5\ m$

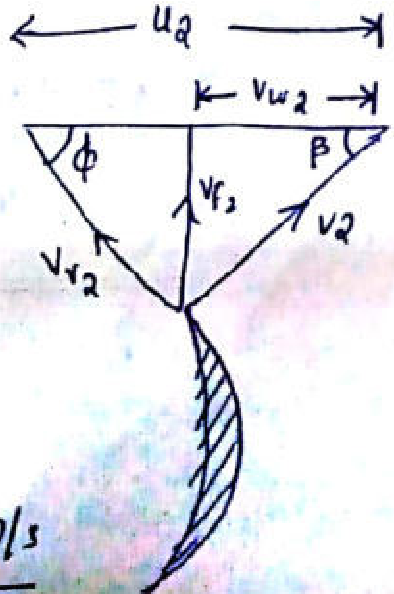
Speed, $N = 1000$ rpm

outer vane angle, $\phi = 30°$
(periphery vane angle)

outer dia, $D_2 = 300\ mm = 0.3\ m$

width, $B_2 = 50mm = 0.05\ m$

$\eta_{mano} = 0.95$



$$\tan\phi = \frac{V_{f2}}{u_2 - V_{w2}}$$

$$u_2 = \frac{\pi D_2 N}{60}$$

$$= \frac{\pi \times 0.3 \times 1000}{60} = \underline{\underline{15.707\ m/s}}$$

$$\eta_{mano} = \frac{gH_m}{V_{w_2} u_2}$$

$$V_{w_2} = \frac{gH_m}{\eta_{mano}\, u_2}$$

$$= \frac{9.81 \times 14.5}{0.95 \times 15.707} = \underline{\underline{9.532}}\ m/s$$

$$\tan\phi = \frac{V_{f_2}}{u_2 - V_{w_2}}$$

$$V_{f_2} = \tan\phi\,(u_2 - V_{w_2})$$

$$= \tan 30\,(15.707 - 9.532)$$

$$= \underline{\underline{3.565\ m/s}}$$

$$Q = \pi D_2 B_2 V_{f_2}$$

$$= \pi \times 0.3 \times 0.05 \times 3.565$$

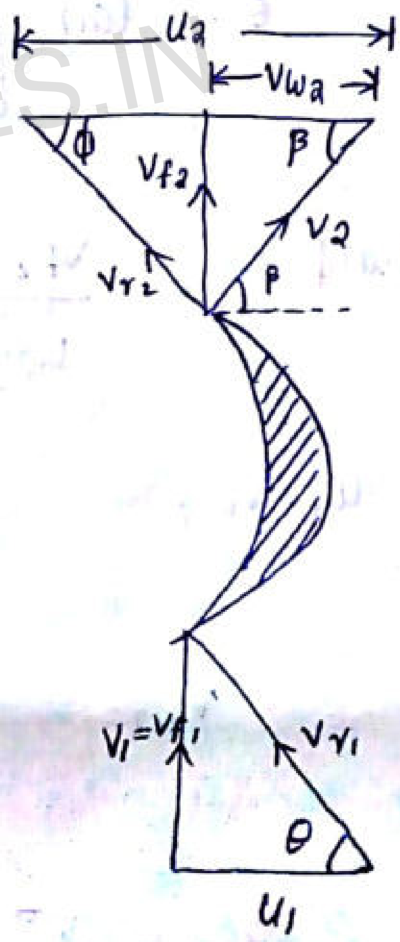
$$= \underline{\underline{0.1679\ m^3/s}}$$

Q) A centrifugal pump having outer dia equal to 2 times the inner dia and running at 1000 rpm works against a head of 40 m. The velocity of flow through the impeller is constant and equal to 2.5 m/s. The vanes are set back at an angle of 40° at outlet. If the outer dia of impeller is 500mm and width at outlet is 50mm Determine

(1) vane angle at inlet

(2) work done by the impeller on water per sec

(3) Manometric efficiency



given

$D_2 = 2 D_1$

$N = 1000$ rpm

$H = 40m$

$V_{f_1} = V_{f_2} = 2.5 \, m/s$

$\phi = 40^\circ$

$D_2 = 500mm = 0.5 \, m$

$B_2 = 50mm = 0.05 m$

$$u_1 = \frac{\pi D_1 N}{60}$$

① vane angle at inlet

$$\tan\theta = \frac{V_{f1}}{u_1}$$

$D_2 = 2D_1$

$D_1 = \frac{D_2}{2}$

$= \frac{0.5}{2} = 0.25$

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.25 \times 1000}{60} = \underline{\underline{13.089 m/s}}$$

$$\tan\theta = \frac{2.5}{13.089}$$

$$\theta = \tan^{-1}\left[\frac{2.5}{13.089}\right] = \underline{\underline{10.813^\circ}}$$

$$\tan\phi = \frac{V_{f2}}{u_2 - V_{w2}}$$

$u_2 = \frac{\pi D_2 N}{60}$

$= \frac{\pi \times 0.5 \times 1000}{60}$

$= \underline{\underline{26.179 m/s}}$

$$u_2 - V_{w2} = \frac{V_{f2}}{\tan\phi}$$

$$V_{w2} = u_2 - \frac{V_{f2}}{\tan\phi}$$

$$= 26.179 - \frac{2.5}{\tan 40}$$

$$= \underline{\underline{23.199 m/s}}$$

Discharge, $Q = \pi D_2 B_2 V_{f_2}$

$= \pi \times 0.5 \times 0.05 \times 2.5$

$= \underline{0.196 \ m^3/s}$

workdone/sec $= \rho Q \ V_{w_2} u_2$

$= 1000 \times 0.196 \times 23.199 \times 26.179$

$= \underline{119036.017 \ Nm/s}$

$$\eta_{mano} = \frac{g H_m}{V_{w_2} u_2}$$

$$= \frac{9.81 \times 40}{23.199 \times 26.179} = 0.64 = \underline{64\%}$$

Multi-stage centrifugal Pumps

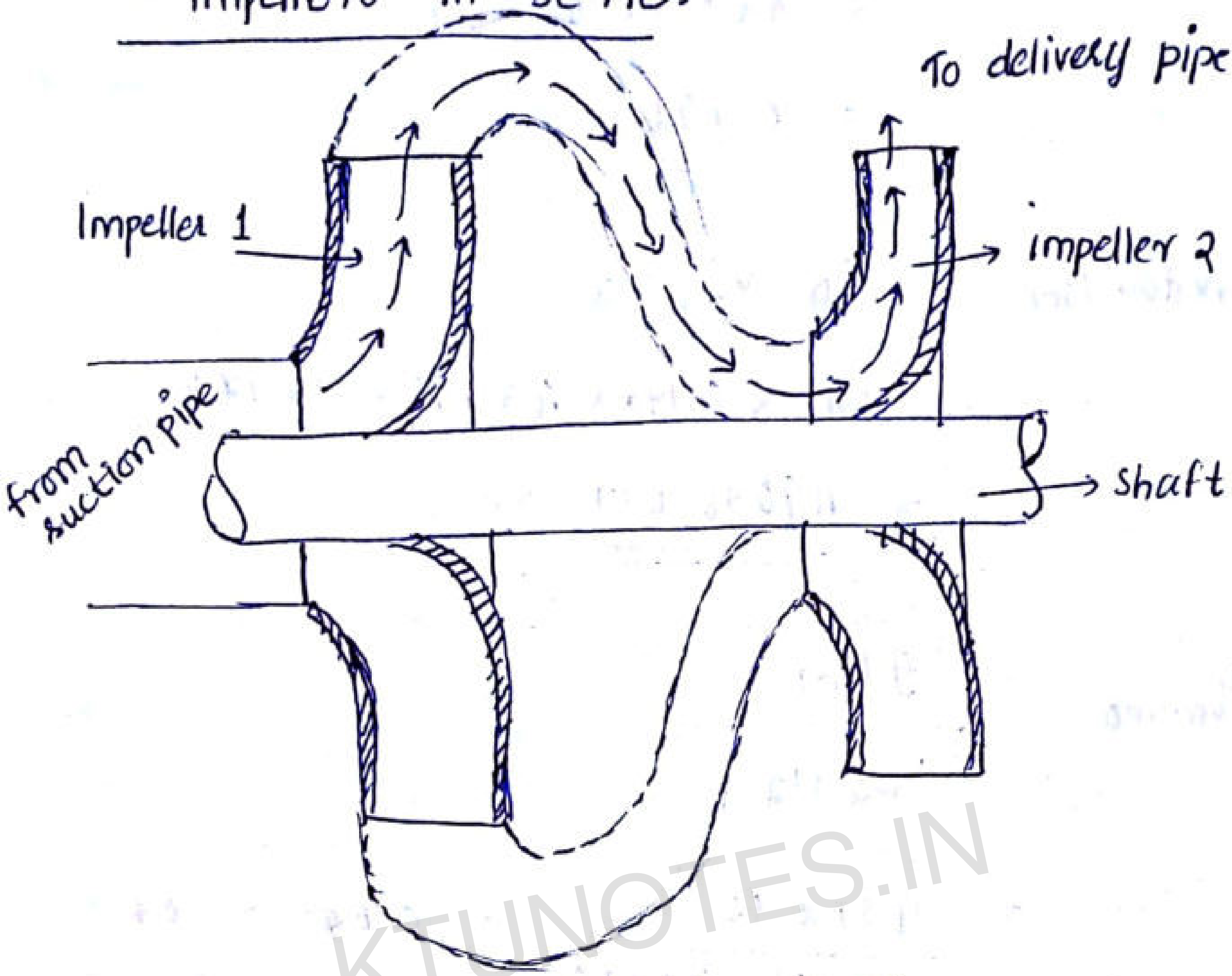
If a centrifugal pump consist of two or more impellers, the pump is called a multi-stage centrifugal pump. The impellers may be mounted on the same shaft or on different shaft

Functions

① To produce high head.

② To discharge a large quantity of liquid

1) Multi- stage centrifugal pump for high heads
- Impellers in series



For developing high head, the no of impellers are mounted in series on the same shaft the water from the suction pipe enters in the first impeller at inlet and discharge at outlet with increased pressure. The water with increased pressure from the first impeller is taken into the inlet of second impeller with the help of a connecting rod At the outlet of first impeller the pressure of water will be pressure of at the outlet of first impeller

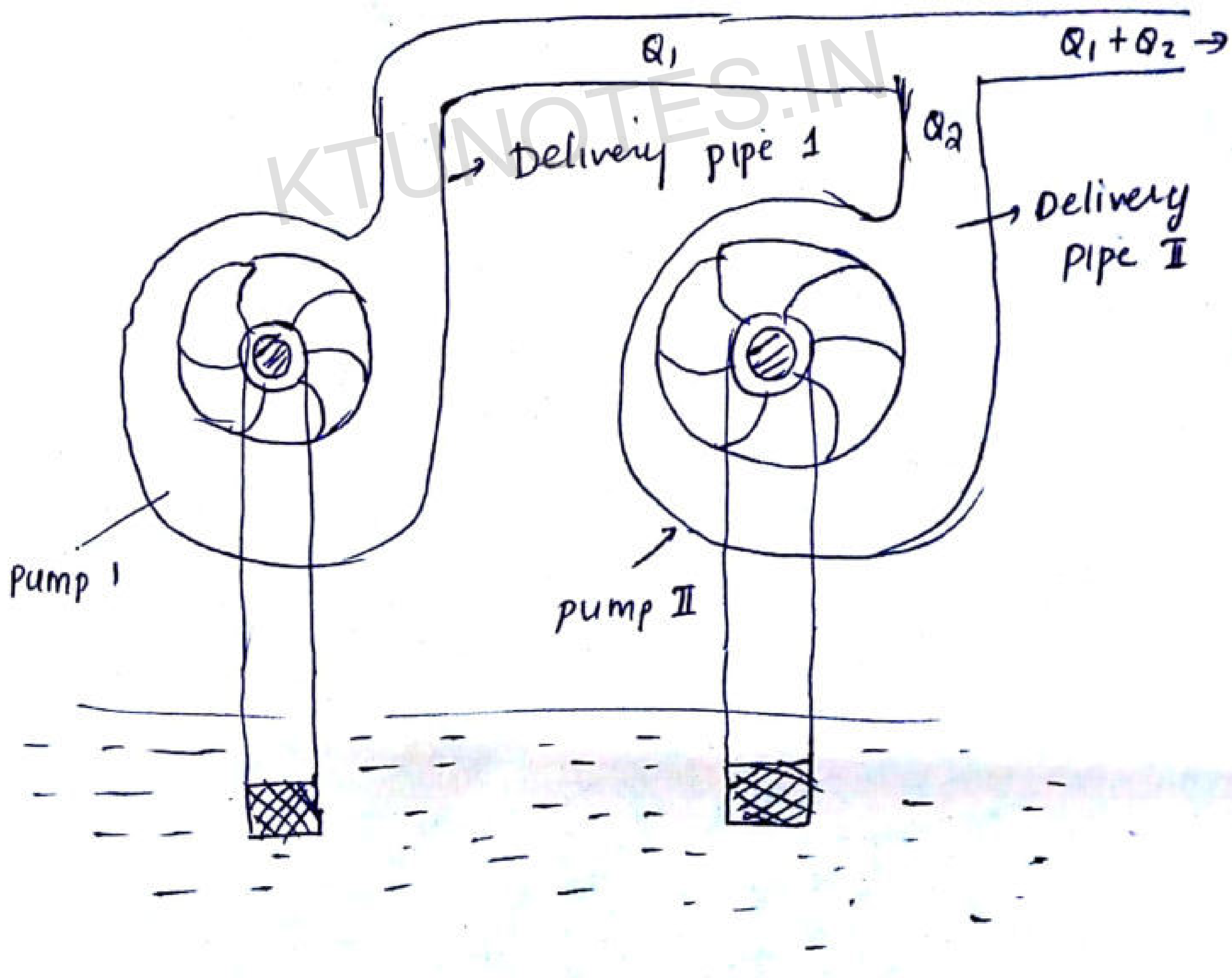
Thus if more impellers are mounted on the same shaft, the pressure at outlet will be increased further.

Then total head developed $= n \times H_m$

where $n =$ no. of identical impellers

$H_m =$ head developed by each impeller

2) Multi-Stage centrifugal pump for large discharge - pumps in parallel



01-03-18

For obtaining large discharge, the pumps should be connected in parallel. each of the pumps lifts the water from a common sump and discharges water to a common pipe to which the delivery pipes of each pump is connected. Each of the pump is working against the same head.

$$\boxed{\text{Total discharge} = n \times Q}$$

where, n = no. of identical pumps arranged in parallel.

Q = discharge from one pump.

Q) A 3 stage centrifugal pump has impellers 40cm in diameter and 2 cm wide at outlet. The vanes are curved back at the outlet at 45° and reduce the circumferential area by 10%. The manometric efficiency is 90% and $\eta_{overall} = 80\%$. Determine the head generated by the pump when running at 1000 rpm and delivering 50 l/sec. What should be the shaft horse power.

$$H_m = \frac{V_{w_2} u_2}{g} - \text{losses}$$

given

$n = 3$

$D_2 = 40\text{cm} = 0.4\text{ m}$
$B_2 = 2\text{cm} = 0.02\text{ m}$ } outlet

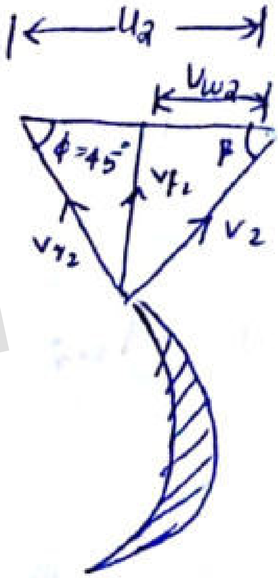
outlet vane angle, $\phi = 45°$

$\eta_{mano} = 90\% = 0.9$

$\eta_{over} = 80\% = 0.8$

$N = 1000$ rpm.

$Q = 50$ lit/sec $= 0.05\text{ m}^3/\text{s}$



① Head generated (H_m)

$$\eta_{mano} = \frac{g H_m}{V_{w_2} u_2}$$

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.4 \times 1000}{60} = \underline{\underline{20.943 \text{ m/s}}}$$

$$\tan\phi = \frac{V_{f_2}}{u_2 - V_{w_2}}$$

$$Q = \pi D_2 B_2 V_{f_2}$$

$$V_{f_2} = \frac{Q}{\pi D_2 B_2}$$

reduction in area by 10%

Area = 100 − 10
= 90

Area = $D_2 B_2$
reduction in 10%
so Area = $0.9 D_2 B_2$

$$Q = 0.9 \times \pi D_2 B_2 V_{f2}$$

$$V_{f2} = \frac{Q}{0.9 \times \pi \times D_2 \times B_2}$$

$$= \frac{0.05}{0.9\pi \times 0.4 \times 0.02} = \underline{\underline{2.210 \text{ m/s}}}$$

$$\tan\phi = \frac{V_{f2}}{u_2 - V_{w2}}$$

$$u_2 - V_{w2} = \frac{V_{f2}}{\tan\phi}$$

$$V_{w2} = u_2 - \frac{V_{f2}}{\tan\phi}$$

$$= 20.943 - \frac{2.210}{\tan 45}$$

$$= \underline{\underline{18.73}} \text{ m/s}$$

$$H_m = \frac{\eta_{mano} \; V_{w_2} \; u_2}{g}$$

$$= \frac{0.9 \times 18.73 \times 20.943}{9.81}$$

$$= \underline{\underline{35.987 \text{ m}}}$$

Total head developed $= n \times H_m$

$= 3 \times 35.987$

$= \underline{\underline{107.961 \text{ m}}}$.

② Shaft power (S.P)

$$\eta_{overall} = \frac{\text{water power}}{\text{shaft power}}$$

$$S.P = \frac{W.P}{\eta_{over}} = \frac{\rho g Q H_m}{\eta_{over} \times 1000}$$

$$= \frac{1000 \times 9.81 \times 0.05 \times 107.961}{0.8 \times 1000}$$

$$= \underline{\underline{66.19}} \text{ kW}$$

Specific Speed of centrifugal Pump (N_s)

The specific speed of a centrifugal pump is defined as the speed of a geometrically similar pump which would deliver one cubic metre of liquid per sec against a head of 1 m.

02-03-2018

$N_s = N, \quad Q = 1 m^3/s \quad , H_m = 1 m.$

Expression for specific speed of centrifugal pump.

Discharge, $Q = \pi D_2 B_2 V_{f_2}$

$$Q \propto D \times B \times V_f$$

we know $D \propto B$

$$Q \propto D^2 \times V_f \quad — (1)$$

Tangential velocity, $u = \frac{\pi D N}{60}$

$$u \propto DN \quad — (2)$$

$$u = k_u \sqrt{2gH}$$

$$u \propto \sqrt{H_m} \quad — (3)$$

Velocity of flow, $V_f = k_f \times \sqrt{2gH}$

$$V_f \propto \sqrt{H_m} \quad — (4)$$

compairing ② and ③

$$DN \propto \sqrt{H_m}$$

$$D \propto \frac{\sqrt{H_m}}{N} \quad \text{— ⑤}$$

sub ④ and ⑤ in ①

$$Q \propto D^2 V_{f_2}$$

$$\propto \left[\frac{\sqrt{H_m}}{N}\right]^2 \cdot \sqrt{H_m}$$

$$\propto \frac{H_m^{3/2}}{N^2}$$

$$Q = K \frac{H_m^{3/2}}{N^2} \quad \text{— ⑥}$$

K = constant of propotionality

for getting value of K

$N = N_s$, $Q = 1 m^3/s$, $H_m = 1 m$

sub in ⑥

$$1 = k \frac{1^{3/2}}{N_s^2}$$

$$N_s^2 = k$$

sub the value of k in ⑥

$$Q = \frac{N_s^2 \times H_m^{3/2}}{N^2}$$

$$N_s^2 = \frac{Q N^2}{H_m^{3/2}}$$

$$\boxed{N_s = \frac{N\sqrt{Q}}{H_m^{3/4}}}$$

Minimum Speed for Starting a centrifugal pump.

$$\text{Centrifugal head} = \frac{(\omega R_2)^2}{2g} - \frac{(\omega R_1)^2}{2g}$$

$$= \frac{u_2^2}{2g} - \frac{u_1^2}{2g} \geq H_m.$$

$$\text{The minimum condition} = \boxed{\frac{u_2^2}{2g} - \frac{u_1^2}{2g} = H_m} \quad -①$$

$$u_2 = \frac{\pi D_2 N}{60} \quad -②$$

$$u_1 = \frac{\pi D_1 N}{60} \quad -③$$

$$\eta_{mano} = \frac{g H_m}{V_{w_2} u_2}$$

$$H_m = \eta_{mano} \times \frac{V_{w_2} u_2}{g} \quad -④$$

sub ②, ③, ④ in ①

$$\frac{1}{2g}\left[\frac{\pi D_2 N}{60}\right]^2 - \frac{1}{2g}\left[\frac{\pi D_1 N}{60}\right]^2 = \eta_{mano} \cdot \frac{V_{w_2}}{g} \times \frac{\pi D_2 N}{60}$$

whole divide by $\frac{\pi N}{960}$.

$$\frac{\pi D_2^2 N}{2\times 60} - \frac{\pi D_1^2 N}{2\times 60} = \eta_{mano} \times V_{w_2} \times D_2$$

$$\frac{\pi N}{120}\left[D_2^2 - D_1^2\right] = \eta_{mano} \times V_{w_2} \times D_2$$

$$\boxed{N = \frac{120\, \eta_{mano} \times V_{w_2} \times D_2}{\pi (D_2^2 - D_1^2)}}$$

15-03-118

Q) The diameters of an impeller of a centrifugal pump at inlet and outlet are 30cm and 60cm respectively. The velocity of flow at outlet is 2m/s and the vanes are set back at an angle of 45° at the outlet. Determine the minimum starting speed of pump if $\eta_{mano} = 70\%$.

given

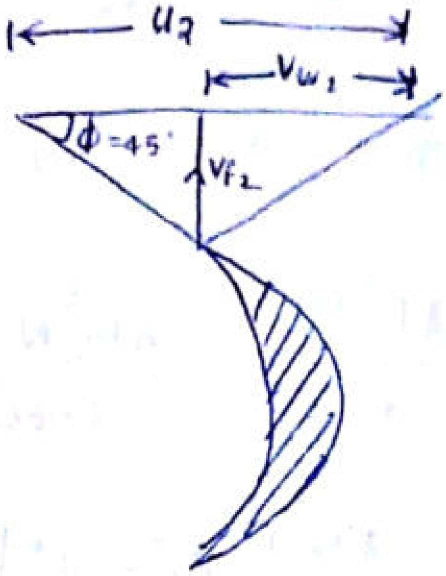
$D_1 = 0.3\,m$

$D_2 = 0.6\,m$

$V_{f_2} = 2\,m/s$

$\phi = 45^\circ$

$\eta_{mano} = 70\% = 0.7$



$$\tan\phi = \frac{V_{f_2}}{u_2 - V_{w_2}}$$

$$u_2 - V_{w_2} = \frac{V_{f_2}}{\tan\phi}$$

$$u_2 - V_{w_2} = \frac{2}{\tan 45} = \underline{\underline{2}} \quad \text{---(1)}$$

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.6 \times N}{60} = 0.031N \quad \text{---(2)}$$

Sub (2) in (1)

$$V_{w_2} = u_2 - \frac{V_{f_2}}{\tan\phi} = 0.031N - 2$$

$$\therefore V_{w_2} = \underline{\underline{0.031N - 2}}$$

$$N = \frac{120 \times 0.7 \times (0.031N - 2) \times 0.6}{\pi (0.6^2 - 0.3^2)}$$

$$= \frac{50.4\,(0.031N - 2)}{0.848}$$

$$\frac{N}{0.031N - 2} = \frac{50.4}{0.848}$$

$$\frac{N}{0.031N - 2} = 59.43$$

$$N = 1.842N - 118.86$$

$$-0.842N = -118.86$$

$$N = \underline{\underline{141.163}} \text{ rpm}$$

Q) The diameters of an impellor of a centrifugal pump at inlet and outlet are 30cm and 60cm respectively. Determine the minimum starting speed of the pump if the pump works against a head of 30m.

<u>given</u>

$D_1 = 0.3m$.

$D_2 = 0.6m$.

$H_m = 30\ m$.

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.3 \times N}{60} = 0.0157\,N$$

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.6 N}{60} = 0.0314\,N$$

$$\frac{u_2^2}{2g} - \frac{u_1^2}{2g} = Hm.$$

$$u_2^2 - u_1^2 = 2gHm.$$

$$(0.0314\,N)^2 - (0.0157N)^2 = 2 \times 9.81 \times 30$$

$$9.8596 \times 10^{-4} N^2 - 2.4649 \times 10^{-4} N^2 = 588.6$$

$$N^2 (7.3947 \times 10^{-4}) = 588.6$$

$$N^2 = 795975.496$$

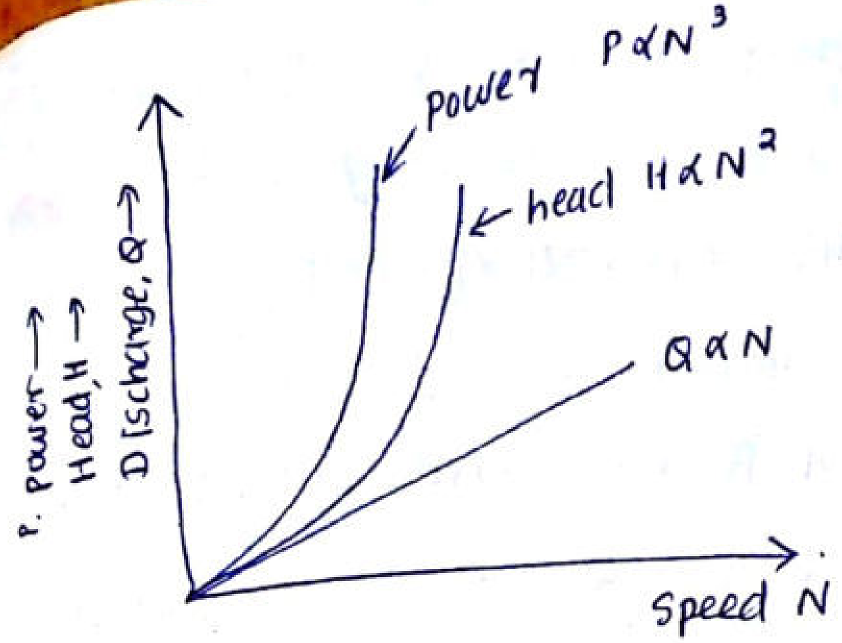
$$N = 892.174 \text{ rpm}$$

Characteristic curves of centrifugal pump

- Main ~~curve~~ characteristic curve
- Operating characteristic curve

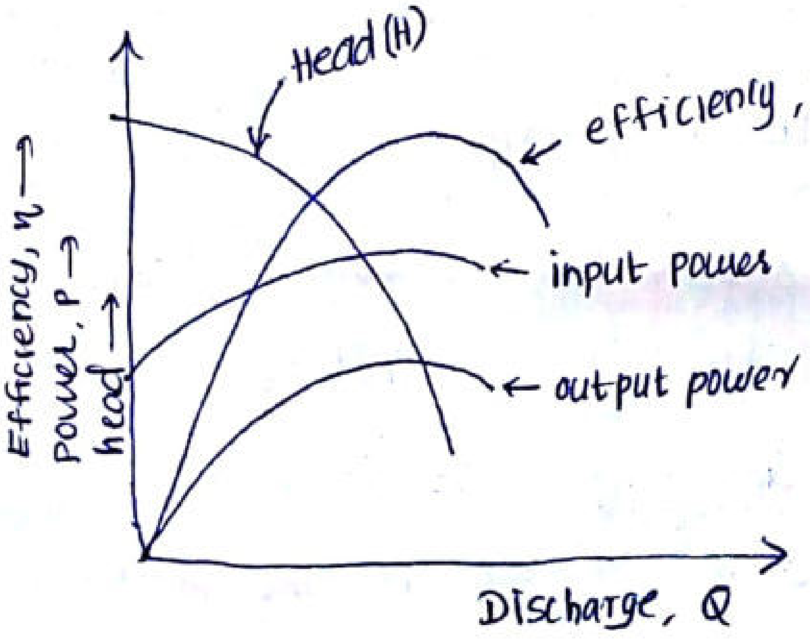
1) Main characteristic curve

The main characteristic curves of a centrifugal pump consist of the variation of manometric head, power and discharge with respect to speed.



- For plotting the curve (manometric head Vs speed), discharge is kept constant.
- For plotting discharge Vs speed, head Hm is const.
- For plotting power Vs speed, head and discharge is kept constant.

2) Operating characteristic curve



If the speed is kept constant and variation of manometric head, power and efficiency with respect to discharge gives the operating or performance characteristics of the pump.

- In the graph, the output power curve will start from origin because when Q = 0, the output power $\rho g Q H = 0$
- The efficiency curve will also start from the origin because when Q = 0 then efficiency will become zero.
- The input power curve shall not pass through the origin because at even zero discharge some power is needed to overcome the mechanical losses

Priming of a centrifugal pump

It is defined as the operation in which the suction pipe, casing of the pipe and the portion of delivery pipe upto delivery valve is completely filled with the liquid to be rised by the pump from outside source before starting the pump. The purpose of priming is to remove the air particles from this parts.

(IMP) Type Number or Shape Number

The dimensionless parameter of specific speed is known as type number or shape number

$$\text{Type number (non-dimensional specific speed)} = \frac{N\sqrt{Q}}{(gH)^{3/4}}$$

Dimensions of type number

$$= \frac{T^{-1}[L^3 T^{-1}]^{1/2}}{[LT^{-2} \times L]^{3/4}} = \frac{L^{3/2} \cdot T^{-3/2}}{L^{3/2} \cdot T^{-3/2}} = 1$$

(IMP) Significance

Type of impeller	Specific speed	Type number
1) Slow speed radial flow impellers	10-30	0·2-0·4
2) Medium speed radial flow impeller	30-50	0·4-1
3) High speed radial flow impeller	50-80	1-1·5
4) Mixed flow impeller	80-160	1·5-3
5) Axial flow impeller	160-above	above 3

Model testing of centrifugal pumps

Before manufacturing large sized pumps their models which are in complete similarity with the actual pumps are made. The complete similarity between model and prototype will exist if the following conditions are satisfied

(1) The specific speed of model should be equal to specific speed of proto type

$$(N_s)_m = (N_s)_p$$

$$\left(\frac{N\sqrt{Q}}{H^{3/4}}\right)_m = \left(\frac{N\sqrt{Q}}{H^{3/4}}\right)_p$$

Tangential velocity, $u = \frac{\pi D N}{60}$

$$u \propto DN \quad \text{—} ①$$

$$u = k_u \sqrt{2gH_m}$$

$$u \propto \sqrt{H_m} \quad \text{—} ②$$

compairing ① and ②

$$\sqrt{H_m} \propto DN$$

$$\frac{\sqrt{H_m}}{DN} = \text{constant}$$

$$\frac{H_m}{D^2 N^2} = \text{constant (head coefficient)}$$

$$\left[\frac{H_m}{D^2N^2}\right]_m = \left[\frac{H_m}{D^2N^2}\right]_p$$

$$\text{Head number} = \frac{gH_m}{D^2N^2}$$

<u>Flow co-efficient or Flow number</u>.

$$\text{Discharge, } Q = \pi D B V_f$$

$$Q \propto D B V_f$$

$$Q \propto D^2 \times V_f \quad \because D \propto B$$

$$V_f \propto U \propto DN \qquad U = \frac{\pi D N}{60}$$

$$Q \propto D^2 DN$$

$$\propto D^3 N$$

$$\frac{Q}{D^3N} = \text{constant (flow coefficient)}$$

$$\left[\frac{Q}{D^3N}\right]_m = \left[\frac{Q}{D^3N}\right]_p$$

$$\frac{Q}{D^3N} = \text{Flow number}$$

05-03-'18

Power coefficient and Power Number

$$\eta_{overall} = \frac{\text{water power}}{\text{shaft power}} = \frac{\rho g Q H_m}{P}$$

$$P = \frac{\rho g Q H_m}{\eta_{overall}}$$

$$P \propto Q H_m \quad \text{— ①}$$

But we know, $Q \propto D^3 N$ — ②

$H_m \propto D^2 N^2$ — ③

Sub ② and ③ in ①

$$P \propto D^3 N \cdot D^2 N^2$$

$$\propto D^5 N^3$$

$$\frac{P}{D^5 N^3} = \text{constant}$$

$$\text{Power coefficient} = \frac{P}{D^5 \cdot N^3}$$

The dimensionless parameter of power coefficient is known as power number. It is obtained by dividing power coefficient by density of liquid

$$\text{Power number} = \frac{P}{\rho D^5 N^3}$$

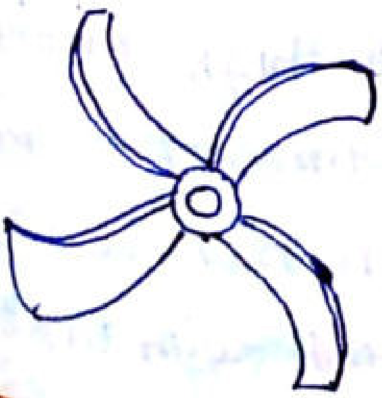
classification of centrifugal pump.

pressure differ
negative press
vacuum press
eye of impeller

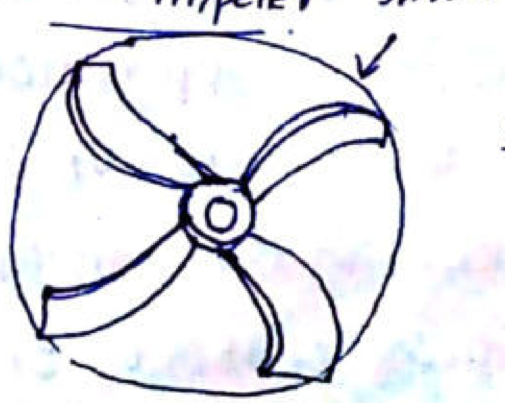
1. Based on type of impeller
   a. closed impeller pump.
   b. semi-open impeller pump.
   c. open impeller pump
2. Based on ~~shape~~ shape and type of casing
   a. volute casing
   b. volute casing with vortex chamber.
   c. Diffuser casing.
3. According to direction of flow through impeller
   a. Radial flow pump
   b. Axial flow pump
   c. Mixed flow pump
4. According to no. of impellers on the shaft
   a. single stage pump
   b. Multistage pump.

1. Based on,

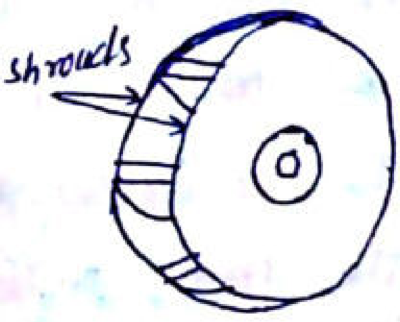
Open impeller



semi-open impeller



closed impeller



Open impeller pump

In open impeller pump, no shroud is provided. the vanes are open in both sides This type of pumps are used where the pump has a very rough (mud+water) duty to perform. This type of pump is used to handle abrasive liquids such as mixture of water and sand or mixture of water and clay etc.

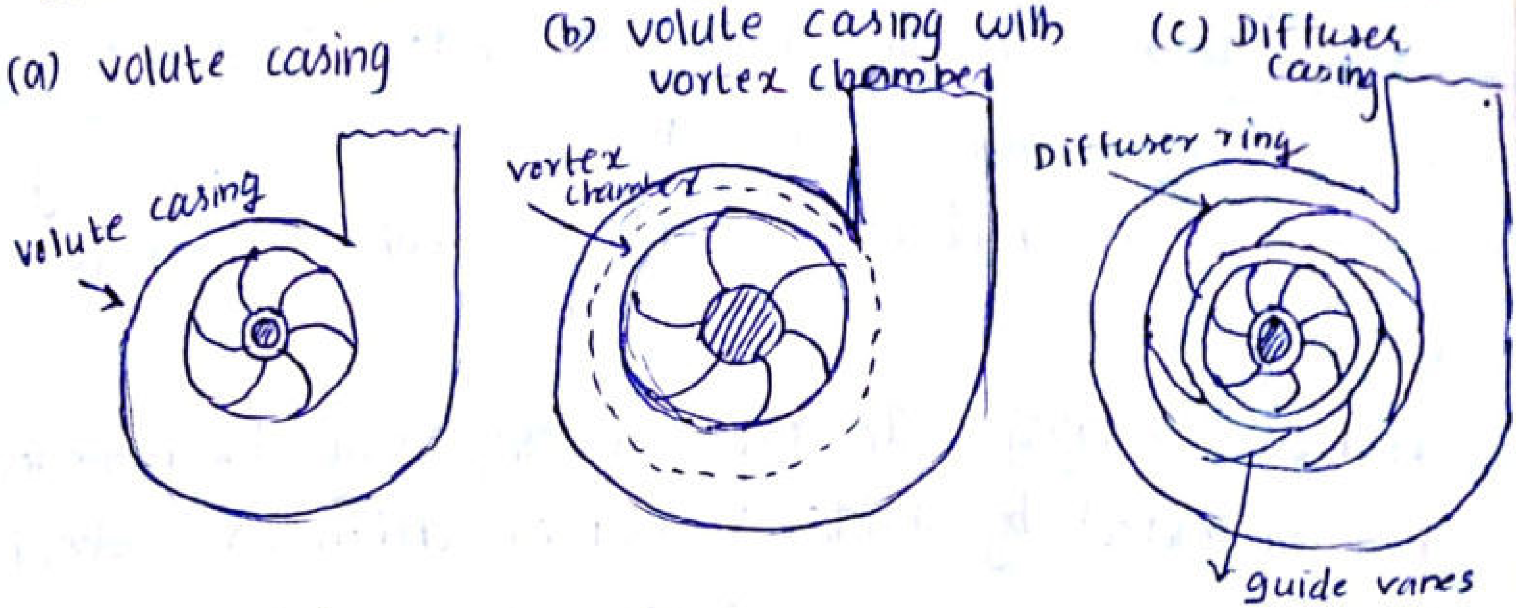
Semi- open impeller

The semi- open type impeller has one shroud only on back side. This type of pump is employed for pumping liquids containing fibrous material such as paper pulp, sewage water, sugar molasses etc

closed impeller pump

ordinary centrifugal pump impellers ~~are~~ are closed type in which the vanes are covered with shrouds on both sides. This arrangement provides smooth passage for the liquids this facility ensures full capacity operation with high efficiency The main disadvantage of closed impeller is that, the friction loss in this impeller is more due to, the more surface contact of liquid with impeller shrouds.

Based on type of casing



Volute casing: In this case, the impeller is surrounded by a spiral casing which is known as volute casing. The area of cross section of volute casing gradually increases towards the delivery pipe. The velocity of the liquids decreases as the area of flow passage increases along the path. This arrangement converts the kinetic energy of the liquid into pressure energy coming out from the casing. The efficiency of this casing is less because large amount of energy is lost due to the formation of eddies in the casing.

Volute casing with vortex chamber: The vortex chamber which is a circular chamber is provided between the impeller and volute casing. In this arrangement, the liquid from the impeller enters the vortex chamber then flows through the volute casing. In this arrangement, the eddy formation is considerably reduced and the efficiency of energy conversion is reduced compared with the volute casing only.

Diffuser casing: In this arrangement, the impeller is surrounded by guide wheel or diffuser wheel consisting of a no. of guide vanes. when water flowing through the diffuser vanes of gradually increasing area; the velocity of flow decreases and kinetic energy of liquid is converted in to pressure energy. The water coming out of the guide vanes passes through the volute casing.

Losses in Centrifugal pump

1) Mechanical losses
2) Hydraulic losses
3) Frictional losses or loss of head due to friction

(1) The mechanical losses are obtained between the shaft and the impeller.

$$\boxed{\text{mechanical losses} = 1 - \text{mechanical efficiency}}$$

(2) Hydraulic losses is occured b/w the impeller and casing

$\boxed{\text{Hydraulic losses} = 1 - \text{hydraulic efficiency}}$

(3) Frictional losses are occured when water flows through the suction pipe and delivery pipe

Bimal ©
05-03-2018

MODULE-4 RECIPROCATING PUMP

Positive displacement pump

Reciprocating pipe

1. Reciprocating pump
2. Diaphragm pump

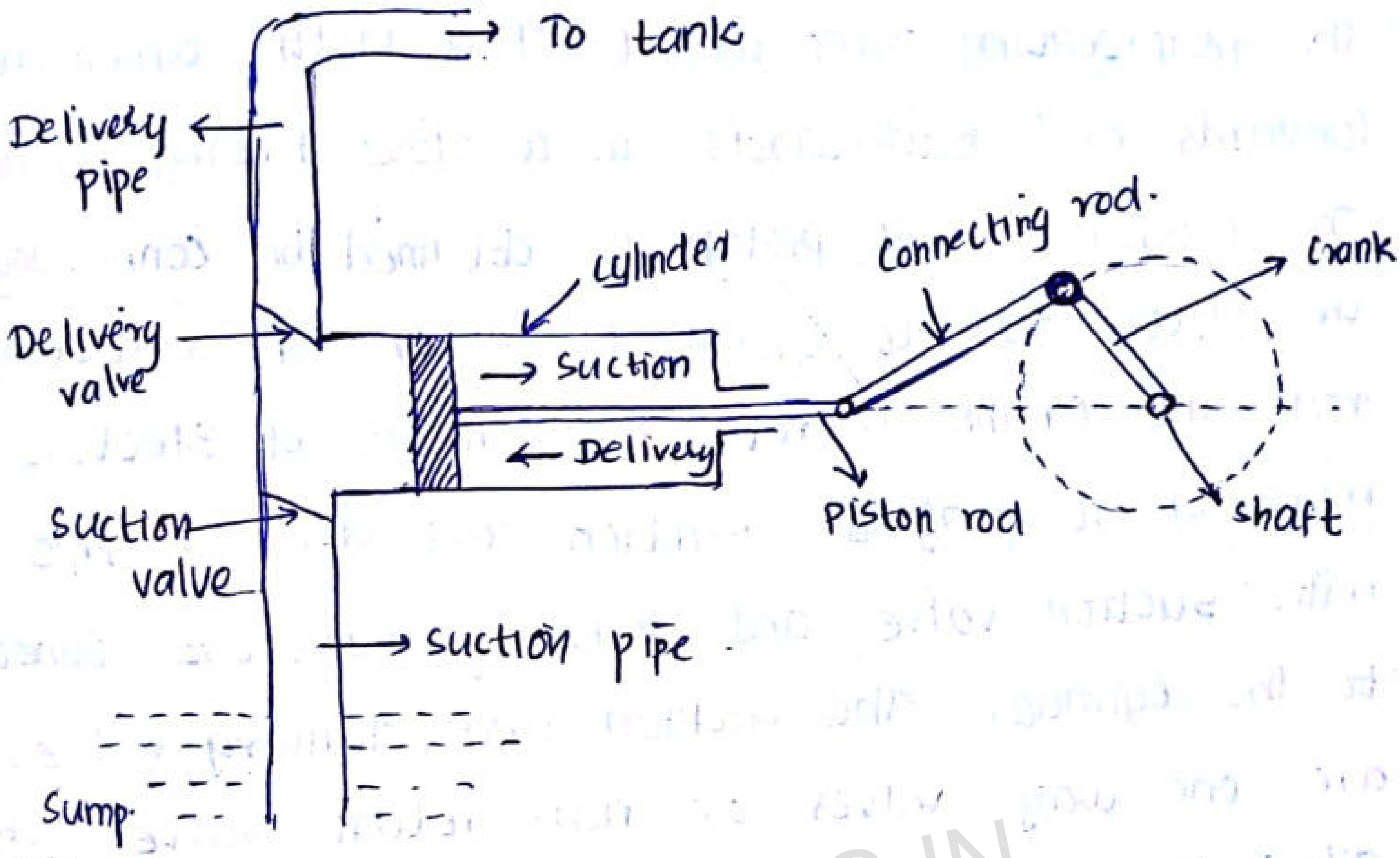
Rotary pipe

1. Gear pump
2. vane pump.
3. Lobe pump.
4. Screw pump

classification

1. Based on water being contact with piston.
   a. single acting
   b. Double acting
2. Based on no.of cylinders
   a. single cylinder pump.
   b. Multi cylinder pump.

Single acting reciprocating pump



12-03-18

Main parts of Reciprocating Pump

1) cylinder with a piston
2) Piston rod
3) connecting rod.
4) crank and crank shaft
5) suction pipe
6) suction valve
7) Delivery valve.
8) Delivery pipe.

Working of Reciprocating pump

The reciprocating pump consist of a piston which moves forwards and backwards in a close fitting cylinder. The movement of piston is obtained by connecting the piston rod to crank by means of connecting rod. The crank is rotated by means of electric motor or IC engine. Suction and delivery pipe with suction valve and delivery valve are connected to the cylinder. The suction and delivery valves are one way valves or non-return valves which allow the water in to flow in one direction only. Suction valve allows water from suction pipe to the cylinder while delivery valve allows water from cylinder to delivery pipe.

Discharge through Reciprocating pump.

D = Diameter of cylinder piston.

$$A = \frac{\pi}{4} D^2$$

L = Length of cylinder

$\therefore$ volume or discharge per stroke $= A \times L = \frac{\pi}{4} d^2 \times L$

speed of crank = N rpm

revolution per minute ⇒ sec ÷ 60

Discharge per sec = $\frac{A \times L \times N}{60}$ m^3/sec

weight of water = $\rho g \times Q$

$= \rho g \times \frac{ALN}{60}$

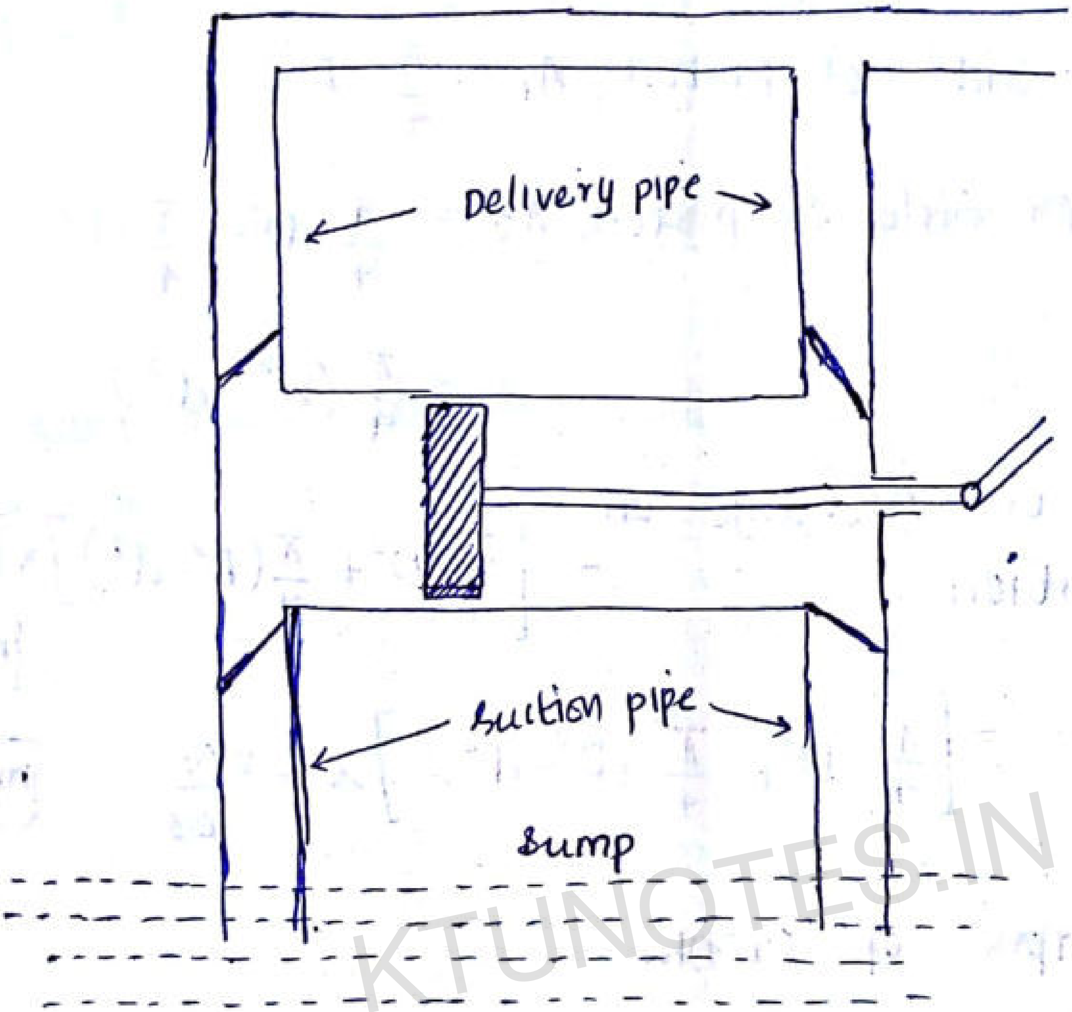
work done/sec = weight of water lifted × Total height of water supplied

$= \rho g \times \frac{ALN}{60} \times (h_s + h_d)$ Nm/s or Watts

Power, P = $\frac{\rho g \times \frac{ALN}{60} \times (h_s + h_d)}{1000}$ kW

12-03-2018

Double acting Reciprocating pump



In the case of double acting pump water is acting on both sides of the piston. In this case 2 suction pipe and 2 delivery pipe are involved. When there is a suction stroke on one side of the piston at the same time there is a delivery stroke on the other side of the piston. thus for one complete revolution of the crank there are 2 delivery strokes and water is delivered to the pipes by the pump during these 2 delivery strokes.

D = diameter of piston.

d = diameter of piston rod

Area of one side of piston, $A_1 = \frac{\pi}{4} \cdot D^2$

Area of other side of piston, $A_2 = \frac{\pi}{4} D^2 - \frac{\pi}{4} d^2$

$$= \frac{\pi}{4}(D^2 - d^2)$$

Total volume or discharge on one revolution $= \left[\frac{\pi}{4} D^2 + \frac{\pi}{4}(D^2 - d^2)\right] \times L$ $\boxed{m^3}$

$$\text{Discharge/sec} = \left[\frac{\pi}{4} D^2 + \frac{\pi}{4}(D^2 - d^2)\right] \times L \times \frac{N}{60} \quad \boxed{m^3/s}$$

N = rpm of crank

If diameter of piston rod is very small compared to diameter of piston ($d <<<< D$), then 'd' can be neglected. then,

$$\text{discharge/sec} = \left[\frac{\pi}{4} D^2 + \frac{\pi}{4} D^2\right] \times \frac{LN}{60}$$

$$Q = 2\left[\frac{\pi}{4} D^2\right] \cdot \frac{LN}{60}$$

$$\boxed{Q = 2A \times \frac{LN}{60}}$$

weight of water lifted/sec $= \rho g \times \frac{2ALN}{60}$

workdone/sec = weight of water lifted $\times$ total height

$$= \rho g \times \frac{2ALN}{60} \times (h_s + h_d)$$

Power required to drive double acting reciprocating pump

$$\boxed{P = \rho g \times \frac{2ALN}{60} \frac{(h_s + h_d)}{1000}}$$

Slip of Reciprocating pump.

The difference b/w theoretical discharge and actual discharge of the pump. The actual discharge of the pump is less than the theoretical discharge is due to various reasons like leakage and head loss due to friction in pipe etc.

mathematically,

$$\boxed{\text{slip} = Q_{th} - Q_{act}}$$

But slip is mostly expressed as percentage of slip.

$$\% \text{ of slip} = \frac{Q_{th} - Q_{act}}{Q_{th}} \times 100$$

$$= \left[1 - \frac{Q_{act}}{Q_{th}}\right] \times 100$$

$$= (1 - C_d) \times 100$$

Where C_d = coefficient of discharge $= \frac{Q_{act}}{Q_{the}}$

Negative slip of Reciprocating pump.

If actual discharge is more than theoretical discharge then the slip of the pump will become negative. In that case the slip of pump is known as negative slip.

The reasons for occuring negative slip are,

① when delivery pipe is short

② when suction pipe is too long.

③ when pump is running at high speed.

Q) A single acting reciprocating pump running at 50 rpm delivers 0.01 m^3/sec of water. The diameter of piston is 200mm and stroke length 400mm. calculate

(1) theoretical discharge

(2) coefficient of discharge

(3) slip and % of slip.

given

$N = 50$ rpm

$Q_{act} = 0.01 \ m^3/s$

$D = 200mm = 0.2m$

$L = 400mm = 0.4 \ m$

$A = \frac{\pi}{4} D^2 = \frac{\pi}{4}(0.2)^2 = 0.0314 \ m^2$ //

$Q_{the} = \frac{ANL}{60} = \frac{0.0314 \times 50 \times 0.4}{60} = 0.0104 \ m^3/s$

$C_d = \frac{Q_{act}}{Q_{th}} = \frac{0.01}{0.0104} = 0.955$ //

$\text{Slip} = Q_{th} - Q_{act} = 0.0104 - 0.01 = 4 \times 10^{-4} \ m^3/s$

$\% \text{ of Slip} = \frac{Q_{th} - Q_{act}}{Q_{th}} \times 100$

$= (1 - C_d) \times 100$

$= (1 - 0.955) \times 100$

$= 4.5 \%$

Q) A double acting reciprocating pump running at 40 rpm is discharging $1 m^3$ of water per minute. The pump has a stroke of 400mm. Dia of piston is 200mm. The delivery and suction head are 20m and 5m respectively. Find the slip of the pump and power required to drive the pump.

given

$N = 40 \text{ rpm}$

$Q_{act} = 1 \; m^3/min = \frac{1}{60} \; m^3/s$

$L = 400mm = 0.4m$

$D = 200mm = 0.2m$

$h_d = 20m$

$h_s = 5m$

$$Q_{th} = \frac{2ALN}{60} = \frac{2 \times 0.0314 \times 0.4 \times 40}{60} = 0.016746 \; m^3/s$$

$$Slip = Q_{th} - Q_{act}$$

$$= 0.016746 - \frac{1}{60} = 8 \times 10^{-5} \; m^3/s$$

$$P = \rho g \times \frac{2ALN}{60} \cdot \frac{h_s + h_d}{1000} = \frac{1000 \times 9.81 \times 0.01674 \times (20+5)}{1000}$$

$$= 4.106 \; kW$$

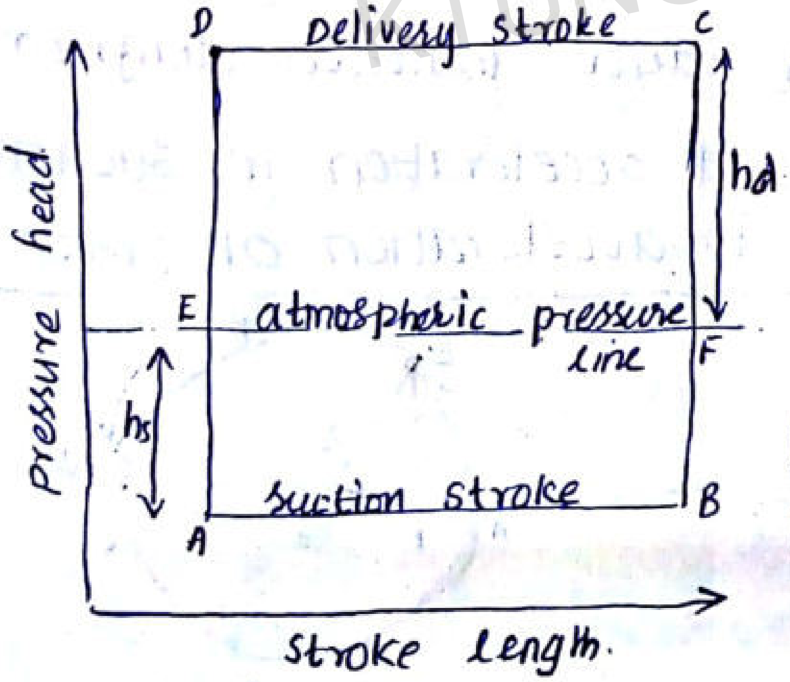
14-03-18

Indicator diagram

The indicator diagram for a reciprocating pump is defined as the graph between the pressure head in the cylinder and the distance travelled by the piston from inner dead centre for one complete revolution of the crank. The pressure head taken as ordinate and stroke length as abscissa.

Ideal indicator diagram

The graph between the pressure head in the cylinder and stroke length of the piston for one complete revolution of the crank under ideal condition is known as ideal indicator diagram.

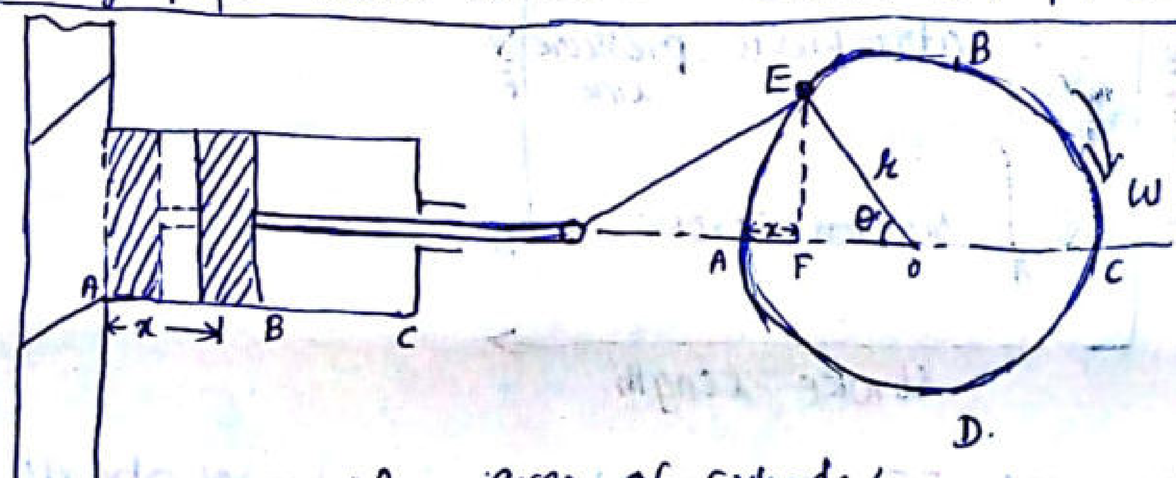


The line EF represents the atmospheric pressure head = 10·3 m of water

During suction stroke, the pressure head in the cylinder is constant and equal to suction head which is below atm. pressure head. The pressure head during suction stroke represented by a horizontal line AB which is below the line EF by a height h_s

During delivery stroke, the pressure head in the cylinder is constant and equal to delivery head and which is above the atm. pressure head. The pressure head during delivery stroke is represented by the line ED which is above the line EF by a height of hd. Thus for one revolution of the crank, the pressure head in the cylinder is represented by the diagram ABCDA. This diagram is known as ideal indicator diagram.

* Variation of velocity and acceleration in suction and delivery pipe due to acceleration of piston.



A = area of cylinder

V = velocity of piston

a = area of pipe

when crank starts rotating, the piston moves forward and backward in the cylinder at extreme left and right position of the piston in cylinder the velocity of piston is zero. The velocity of piston at center (B) is maximum. At the begining of each stroke the piston have the maximum acceleration and at the end of each stroke. piston have maximum retardation. The water in the cylinder in contact with the piston and the water in suction pipe and delivery pipe will have the same acceleration and retardation at the begining and end of the stroke. This accelerative and retarding head will change the pressure head in the cylinder

Crank angle, $\theta = \omega t$

$$x = AF = AO - FO$$

$$= r - r\cos\theta$$

$$= r - r\cos\omega t$$

$$V = \frac{dx}{dt} = \frac{d}{dt}(r - r\cos\omega t)$$

$$= 0 - (r \times -\sin\omega t \cdot \omega)$$

$$= \omega r \sin\omega t$$

Applying continuity equation,

$$A \times V = a \times v$$

$$v = \frac{A \times V}{a}$$

$$v = \frac{A}{a} \times \omega r \sin \omega t$$

acceleration ratio $= \frac{dv}{dt}$

$$= \frac{d}{dt} \frac{A}{a} (\omega r \sin \omega t)$$

$$= \frac{A}{a} \omega r \cos \omega t \; \omega$$

$$= \frac{A}{a} \omega^2 r \cos \omega t$$

19-03-2018

Acceleration of water in pipe $= \frac{A}{a} \omega^2 r \cos \omega t$

Force required = mass x acceleration

$$= \rho \times (a \times l) \times \frac{A}{a} \omega^2 r \cos \omega t$$

pressure intensity inside the pipe $(\;) = \frac{\text{Force}}{\text{area of pipe}}$

$$= \frac{\rho \cancel{a} l \times \frac{A}{a} \omega r^2 \cos \omega t}{\cancel{a}}$$

pressure $= \rho l \times \frac{A}{a} \omega^2 r \cos\omega t$

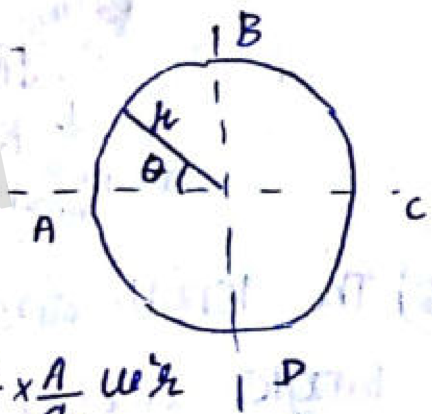
Pressure, $P = \rho g h$

$$h_a = \frac{\text{Pressure}}{\rho g} = \frac{\rho l \times \frac{A}{a} \times \omega^2 r \cos\omega t}{\rho g}$$

pressure head due to acceleration, $h_a = \frac{l}{g} \times \frac{A}{a} \omega^2 r \cos\omega t$

we know, $\omega t = \theta$

$$\boxed{h_a = \frac{l}{g} \times \frac{A}{a} \cdot \omega^2 r \cos\theta}$$



<u>Suction stroke</u>

At A, when $\theta = 0°$, $\cos 0 = 1$, $h_{a_s} = \frac{l_s}{g} \times \frac{A}{a_s} \cdot \omega^2 r$
(starting of suction stroke)

At B, when $\theta = 90°$, $\cos 90 = 0$, $h_{a_s} = 0$
(middle of suction stroke)

At C, when $\theta = 180°$, $\cos 180 = -1$, $h_{a_s} = -\frac{l_s}{g} \times \frac{A}{a_s} \omega^2 r$
(ending of suction stroke)

$\omega \Rightarrow$ Angular velocity

$r \Rightarrow$ crank radius

<u>Delivery stroke</u>

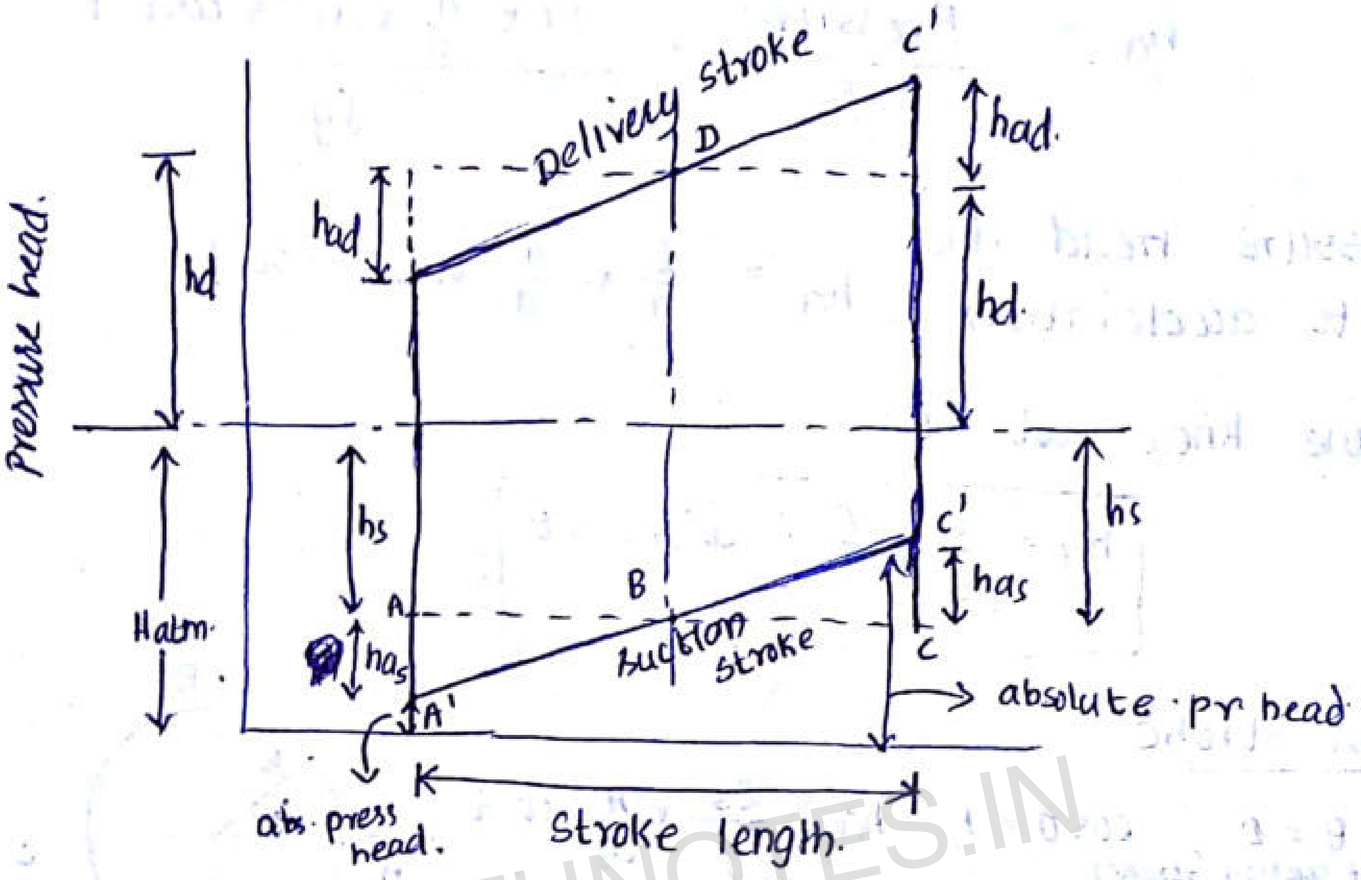
At C, when $\theta = 0°$, $\cos 0 = 1$, $h_{a_d} = \frac{l_d}{g} \times \frac{A}{a_d} \times \omega^2 r$

At D, when $\theta = 90°$, $\cos 90 = 0$, $h_{a_d} = 0$

At A, when $\theta = 180°$, 360°, $\cos 180 = -1$, $h_{a_d} = -\frac{l_d}{g} \times \frac{A}{a_d} \times \omega^2 r$

$(h_a)_{max} = \frac{l}{g} \times \frac{A}{a} \times \omega^2 r$ (both suction and delivery stroke)
(when $\theta = 0$)

Effect of acceleration in suction and Delivery pipes on indicator diagram.



Q) The length and diameter of a suction pipe of a single acting reciprocating pump are 5m and 10cm respectively. The pump has a plunger (piston) of dia 15cm and stroke length of 35cm. The centre of the pump is 3m above the water surface in sump. The atmospheric pressure head is 10.3m of water and pump is running at 35 rpm. Determine

(1) pressure head due to acceleration at the begining of suction stroke

(2) maximum pressure head due to acceleration.

(3) Pressure head in the cylinder at the begining and end of suction stroke

given

$l_s = 5m.$

$d_s = 10cm. = 0.1 m.$

$a_s = \frac{\pi}{4} d_s^2 = \frac{\pi}{4} \times (\cdot 1)^2 = \cancel{78.539\ m^2}\ 7.853 \times 10^{-3} m^2$

Dia. of piston, $D = 15cm = 0.15m.$

Area of piston, $A = \frac{\pi}{4} D^2 = \frac{\pi}{4} \times 0.15^2 = 0.0176 m^2$

stroke length, $L = 35cm = 0.35m.$

$H_{atm} = 10.3m.$

Speed of pump, $N = 35$ rpm.

(1) $h_{as} = \frac{l_s}{g} \times \frac{A}{a_s} \times \omega^2 r$

r = Crank radius $= \frac{L}{2}$

$= \frac{0.35}{2} = 0.175 m$

$\omega = \frac{2\pi N}{60}$

$= \frac{2\pi \times 35}{60}$

$= 3.665\ m/s.$

$h_{as} = \frac{5}{9.81} \times \frac{0.0176}{\cancel{78.539}\ 7.853 \times 10^{-3}} \times 3.665^2 \times 0.175$

$= 2.685\ m$

② $(h_a)_{max} = \frac{l}{g} \times \frac{A}{a} \times \omega^2 r$

$$= \frac{5}{9.81} \times \frac{0.0176}{7.853 \times 10^{-3}} \times 3.665^2 \times 0.175$$

$$= \underline{\underline{2.685\ m}}$$

③ Pressure head at begining $= h_s + h_{as}$

$$= 3 + 2.685$$

$$= \underline{\underline{5.685\ m}}$$

Absolute pressure head at the begining $= 10.3 - (h_s + h_{as})$

$$= 10.3 - 5.685$$

$$= \underline{\underline{4.615\ m}}$$

Pressure head at end $= h_s - h_{as}$

$$= 3 - 2.685$$

$$= \cancel{1.685\ m} = 0.315\ m$$

Absolute pressure head at the end $= 10.3 - (h_s - h_{as})$

$$= 10.3 - 0.315$$

$$= 9.985\ m$$

21-03-'18

Effect of friction in Suction and Delivery pipes.

We know, velocity in pipe, $v = \frac{A}{a} \omega r \sin\omega t$

$= \frac{A}{a} \omega r \sin\theta$

But head loss due to friction, $h_f = \frac{4flv^2}{2gd}$

$= \frac{4fl}{2gd}\left(\frac{A}{a} \omega r \sin\omega t\right)^2$

$\therefore h_{fs} = \frac{4fl_s}{2gd_s}\left(\frac{A}{a_s} \omega r \sin\omega t\right)^2$

$h_{fd} = \frac{4fl_d}{2gd_d}\left(\frac{A}{a_d} \omega r \sin\omega t\right)^2$

When $\theta = 0$, $\sin 0 = 0$ (begining of stroke)

$\therefore h_{fs} = h_{fd} = 0.$

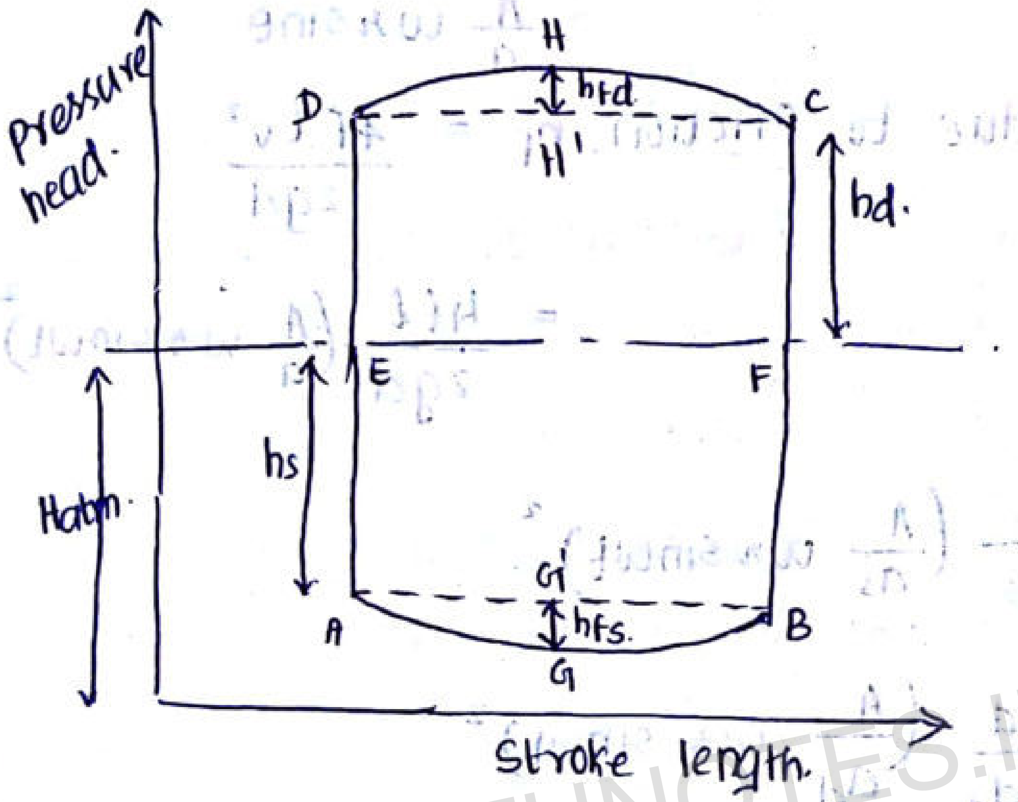
When $\theta = 90°$, $\sin 90 = 1$ (middle of stroke).

$\therefore h_{fs} = \frac{4fl_s}{2gd_s}\left(\frac{A}{a_s} \omega r\right)^2$

$h_{fd} = \frac{4fl_d}{2gd_d}\left(\frac{A}{a_d} \omega r\right)^2$

At the end of stroke, $\theta = 180^\circ$, $\sin 180 = 0$

$$\therefore h_{fs} = h_{fd} = 0$$



Area $AG_1B = AB \times \frac{2}{3} GG'$

$= L \times \frac{2}{3} \times h_{fs}$

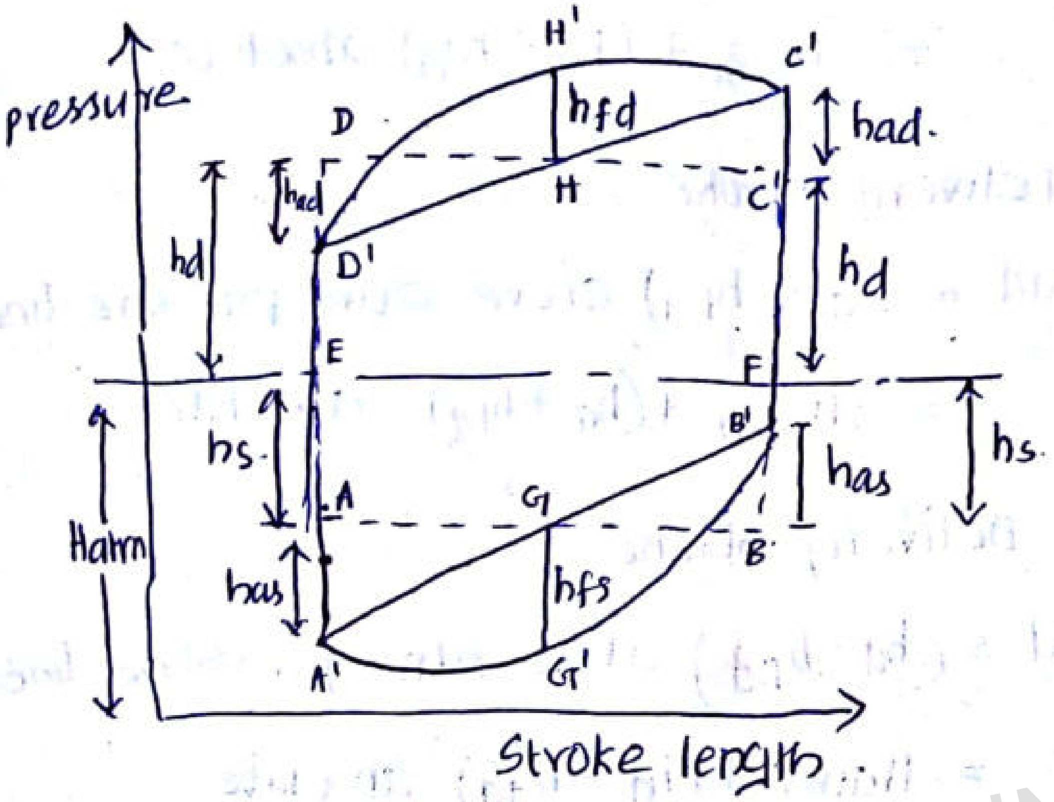
Area $CHD = CD \times \frac{2}{3} HH'$

$= L \times 2/3 \times h_{fd}$

Area of ideal indicator diagram.

$= L \times (h_s + h_d)$

Combined effect of acceleration and friction in suction and delivery pipes



At the begining of suction stroke

Pressure head = $(h_s + h_{as})$ below atm. pressure line

$= H_{atm} - (h_s + h_{as})$ absolute.

At middle of suction stroke

Pressure head $= (h_s + h_{fs})$ below atm. pressure line

$= H_{atm} - (h_s + h_{fs})$ absolute.

At the end of suction stroke

Pressure head $= (h_s - h_{as})$ below atm. pressure line

$= H_{atm} - (h_s - h_{as})$ absolute

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At the begining of Delivery stroke

Pressure head = $(h_d + h_{ad})$ ~~below~~ above atm. pressure line

= $H_{atm} + (h_d + h_{ad})$ absolute

At middle of Delivery stroke,

Pressure head = $(h_d + h_{fd})$ above atm. pressure line

= $H_{atm} + (h_d + h_{fd})$ absolute

At the end of Delivery stroke

Pressure head = $(h_d - h_{ad})$ above atm. pressure line

= $H_{atm} + (h_d - h_{ad})$ absolute

Area of indicator diagram.

= Area $A'B'C'D'$ + Area $A'GB'$ + Area $C'H'D'$

= Area $ABCD + A'B' \times \frac{2}{3} h_{fs} + C'D' \times \frac{2}{3} \times h_{fd}$

= $L \times (h_s + h_d) + AB \times \frac{2}{3} h_{fs} + CD \times \frac{2}{3} \times h_{fd}$.

= $L \times (h_s + h_d) + L \times \frac{2}{3} \times h_{fs} + L \times \frac{2}{3} \times h_{fd}$.

= $L\left(h_s + h_d + \frac{2}{3} h_{fs} + \frac{2}{3} h_{fd}\right)$

workdone by the pump $\propto$ Area of indicator diagram.

$$\propto L\left(h_s + h_d + \frac{2}{3}h_{fs} + \frac{2}{3}h_{fd}\right)$$

$$= k\,L\left(h_s + h_d + \frac{2}{3}h_{fs} + \frac{2}{3}h_{fd}\right)$$

where, k = constant of propotionality

$$= \frac{\rho g A N}{60} \rightarrow \text{for single acting pump.}$$

$$= \frac{2\rho g A N}{60} \rightarrow \text{for double acting pump.}$$

Q) The diameter and stroke length of a single acting reciprocating pump are 12cm and 20cm respectively. The lengths of suction and delivery pipes are 8m and 25m respectively and their diameters are 7.5cm. If the pump is running at 40 rpm and suction and delivery heads are 4m and 14m respectively. Find the pressure head in the cylinder

(1) At the begining of suction and delivery stroke

(2) At the middle of suction and delivery stroke

(3) At the end of suction and delivery stroke

Take atm. pressure head = 10.3 m of water and f = 0.009 for both pipes

given

Dia of piston, $D = 12\,cm = 0.12\,m$

Stroke length, $L = 20\,cm = 0.2\,m$.

Length of suction pipe, $l_s = 8\,m$.

Length of delivery pipe, $l_d = 25\,m$.

Dia of suction and delivery pipe, $d_s, d_d = 7.5\,cm = 0.075\,m$

Speed, $N = 40$ rpm.

suction head, $h_s = 4\,m$.

delivery head, $h_d = 14\,m$.

$H_{atm} = 10.3\,m$.

$f = 0.009$.

(1) At the begining of suction stroke.

Pressure head $= H_{atm} - (h_s + h_{as})$ (absolute)

$\therefore h_{as} = \frac{l_s}{g} \times \frac{A}{a_s} \times \omega^2 r \cos\theta$

$A = \frac{\pi}{4} D^2 = \frac{\pi}{4} \times 0.12^2 = 0.0113\ m^2$ //

$a_s = \frac{\pi}{4} d_s^2 = \frac{\pi}{4} \times 0.075^2 = 4.417 \times 10^{-3}\ m^2$ //

$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 40}{60} = 4.188$ m/s //

r - Crank radius = $\frac{L}{2} = \frac{0.2}{2} = 0.1\,m$

$\theta = 0$ (at the begining)

~~Pressure head~~ =

$$h_{as} = \frac{l_s}{g} \times \frac{A}{a_s} \times \omega^2 r \cos\theta$$

$$= \frac{8}{9.81} \times \frac{0.0113}{4.417 \times 10^{-3}} \times 4.188^2 \times 0.1 \times \cos 0$$

$$= \underline{\underline{3.659\,m}}$$

Pressure head = $H_{atm} - (h_s + h_{as})$ absolute

$$= 10.3 - (4 + 3.659)$$

$$= \underline{\underline{2.641\,m}}$$

(2) At the middle of suction stroke

Pressure head = $H_{atm} - (h_s + h_{fs})$

$$h_{fs} = \frac{4 f l_s}{2 g d_s} \left(\frac{A}{a_s} \omega r \sin\theta\right)^2$$

$$= \frac{4 \times 0.009 \times 8}{2 \times 9.81 \times 0.075} \left[\frac{0.0113}{4.417 \times 10^{-3}} \times 4.188 \times 0.1 \times \sin 90\right]^2$$

$$= \underline{\underline{0.224\,m}}$$

pressure head $= H_{atm} - (h_s + h_{fs})$

$= 10.3 - (4 + 0.224)$

$= \underline{\underline{6.076\ m}}$

(3) At the end of suction stroke

Pressure head $= H_{atm} - (h_s - h_{as})$

$= 10.3 - (4 - 3.659)$

$= \underline{\underline{9.959\ m}}$

(4) At the begining of delivery stroke.

pressure head $= H_{atm} + (h_d + h_{ad})$

$$h_{ad} = \frac{l_d}{g} \times \frac{A}{a_d} \times w^2 r \cos\theta$$

$$a_d = \frac{\pi}{4} d_d^2 = \frac{\pi}{4} \times 0.075^2 = 4.417 \times 10^{-3}\ m^2$$

$$h_{ad} = \frac{25}{9.81} \times \frac{0.0113}{4.417 \times 10^{-3}} \times 4.188^2 \times 0.1 \times \cos 0$$

$= \underline{\underline{11.434\ m.}}$

pressure head = $H_{atm} + [h_d + h_{ad}]$

$= 10.3 + [14 + 11.434]$

$= \underline{\underline{35.734\ m}}$

(5) At the middle of delivery stroke

pressure head = $H_{atm} + [h_d + h_{fd}]$

$$h_{fd} = \frac{4 f l_d}{2 g d_d}\left[\frac{A}{a_d}\, \omega r \sin\theta\right]^2$$

$$= \frac{4 \times 0.009 \times 25}{2 \times 9.81 \times 0.075}\left[\frac{\cancel{0.0113}\ (\pi/4 \times 0.12^2)}{\cancel{4.418\times10^{-3}}\ (\pi/4 \times 0.075^2)} \times 4.188 \times 0.1 \times \sin 90\right]^2$$

$= \cancel{0.2229\ m}\ \ \underline{\underline{0.703\ m}}$ 0.703

Pressure head = $H_{atm} + [h_d + h_{fd}]$

$= 10.3 + [14 + \cancel{0.2229}\ 0.703]$

$= \cancel{25.2229}\ \ \underline{\underline{25.003\ m}}$

(6) At the end of delivery stroke

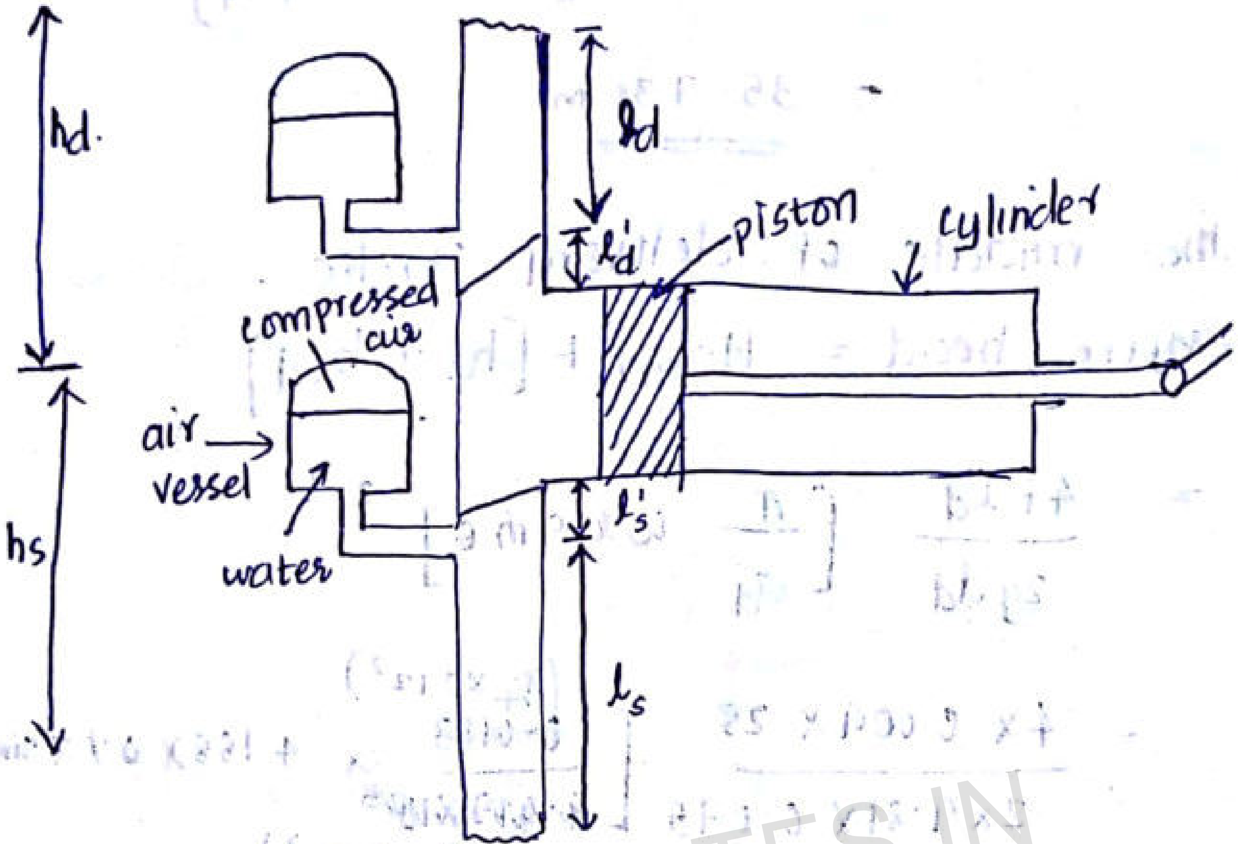
pressure head = $H_{atm} + [h_d - h_{ad}]$

$= 10.3 + [14 - 11.434]$

$= \underline{\underline{12.866\ m.}}$

26-03-'18 (IMP)

Air Vessel :



An air vessel is a closed chamber containing compressed air in the top portion and liquid at the bottom of the chamber. At the base of the chamber, there is an opening through which the liquid may flow into the vessel or out of the vessel. When the liquid enters the vessel, the air gets compressed further and when the liquid flows out of the vessel, the air will expand in the chamber.

An air vessel is fitted to the suction pipe and delivery pipe at a point close to the cylinder of a single acting reciprocating pump executes the following functions

(1) To obtain a continous supply of liquid at uniform rate

(2) To save a considerable amount of work in overcoming the frictional resistance in the suction and delivery pipe.

(3) To run the pump at a high speed without separation.

In the figure,

l_s = length of suction pipe below air vessel

l_s' = length of suction pipe b/w cylinder & air vessel

l_d' = length of delivery pipe b/w cylinder & air vessel

l_d = length of delivery pipe beyond air vessel.

For single acting pump,

$\boxed{Q = AV}$

Discharge, $Q = \frac{ALN}{60}$

Mean velocity, $\bar{v} = \frac{\text{Discharge}}{\text{area of pipe}} = \frac{Q}{a}$

$$= \frac{ALN}{60 \times a}$$

$$= \frac{A}{60a} \times 2r \times \frac{60\omega}{2\pi}$$

$$\boxed{\bar{V} = \frac{A}{a} \cdot \frac{\omega r}{\pi}}$$

$\omega = \frac{2\pi N}{60}$

$N = \frac{60\omega}{2\pi}$

$L = 2r$

r = crank radius

work saved by fitting air vessel.

work done by the pump per stroke against friction

w_1 = area of parabola

$= 2/3 \times$ base $\times$ height

$= 2/3 \times L \times h_f$

But $h_f = \frac{4fl}{2gd}\left(\frac{A}{a}\omega r\right)^2$ when $\theta = 90°$

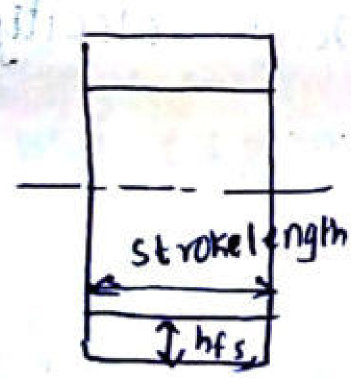
$$\boxed{\therefore W_1 = \frac{2}{3} \times L \times \frac{4fl}{2gd}\left[\frac{A}{a}\omega r\right)^2}$$

work done by air vessel against friction.

W_2 = Area of rectangle

$=$ base $\times$ height

$= L \times h_{fs}$



but $h_{fs} = \frac{4fl}{2gd} (\bar{V})^2$

$$= \frac{4fl}{2gd} \left(\frac{A}{a} \frac{\omega r}{\pi}\right)^2$$

$$\therefore W_2 = L \times \frac{4fl}{2gd} \left[\frac{A}{a} \frac{\omega r}{\pi}\right]^2$$

$$W_2 = \frac{1}{\pi^2} \times L \times \frac{4fl}{2gd} \left(\frac{A}{a} \omega r\right)^2$$

work saved per stroke $= W_1 - W_2$

$$= \frac{2}{3} \cdot L \times \frac{4fl}{2gd} \left[\frac{A}{a} \omega r\right]^2 - \frac{1}{\pi^2} \cdot L \cdot \frac{4fl}{2gd} \left(\frac{A}{a} \omega r\right)^2$$

$$\boxed{W_1 - W_2 = \left[\frac{2}{3} - \frac{1}{\pi^2}\right] \times L \times \frac{4fl}{2gd} \left[\frac{A}{a} \omega r\right]^2}$$

IMP

Percentage of work saved per stroke $= \frac{W_1 - W_2}{W_1} \times 100$

$$= \frac{\left[\frac{2}{3} - \frac{1}{\pi^2}\right] \times L \times \frac{4fl}{2gd} \left(\frac{A}{a} \omega r\right)^2}{\frac{2}{3} \times L \times \frac{4fl}{2gd} \left(\frac{A}{a} \cdot \omega r\right)^2} \times 100$$

$$= \frac{\left[\frac{2}{3} - \frac{1}{\pi^2}\right]}{2/3} \times 100 = \underline{\underline{84.8\%}}$$

IMP **Work saved in double acting Reciprocating pump**

work done against friction without air vessel,

$$W_1 = \frac{2}{3} \times L \times \frac{4fl}{2gd}\left(\frac{A}{a}\,\omega r\right)^2$$

work done against friction with air vessel,

W_2 = Area of rectangle

= Base x height

$= L \times h_f$

$$= L \times \frac{4fl}{2gd}\left[\bar{V}^2\right]$$

But $\bar{V} = \dfrac{\text{Discharge}}{\text{area of pipe}} = \dfrac{2ALN}{60 \times a}$

$$\left\{ \begin{array}{l} \omega = \dfrac{2\pi N}{60} \\ N = \dfrac{60\omega}{2\pi} \\ L = 2r \end{array} \right.$$

$$= \frac{2 \times A \times 2r}{60a} \times \frac{60\omega}{2\pi} = 2\,\frac{A}{a}\,\frac{\omega r}{\pi}$$

$$W_2 = L \times \frac{4fl}{2gd} \times \left[2\,\frac{A}{a}\,\frac{\omega r}{\pi}\right]^2$$

$$= \frac{4}{\pi^2} \times L \times \frac{4fl}{2gd}\left(\frac{A}{a}\,\omega r\right)^2$$

work saved per stroke with air vessel

$= W_1 - W_2$

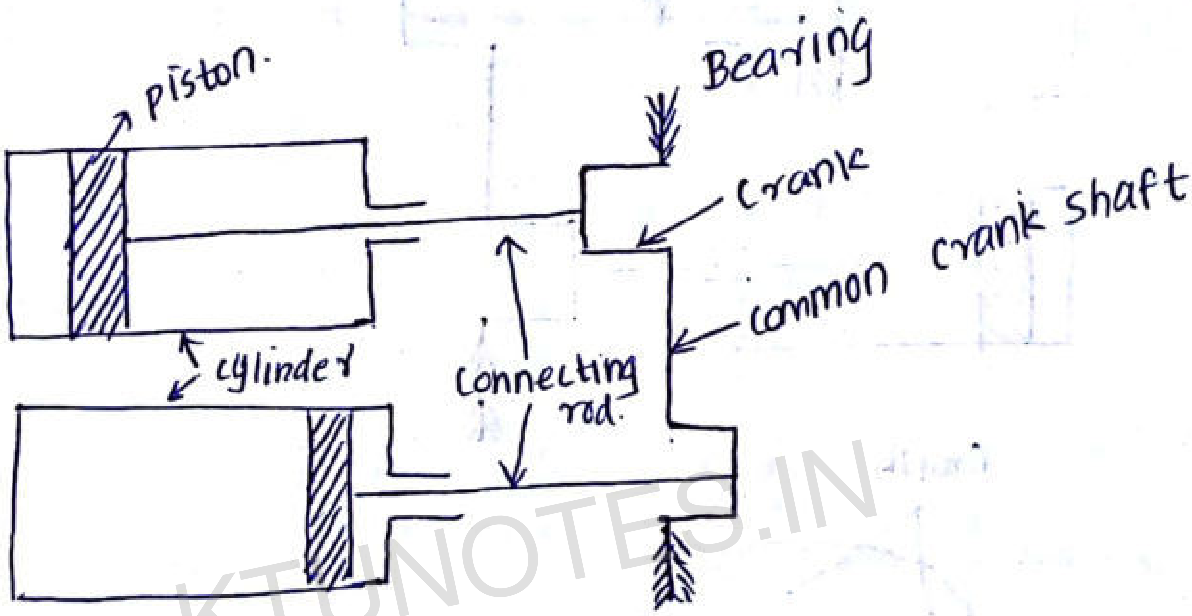
$$= \left(\frac{2}{3} - \frac{4}{\pi^2}\right) L \times \frac{4fl}{2gd}\left(\frac{A}{a}\,\omega r\right)^2$$

percentage of saved per stroke

$$= \frac{W_1 - W_2}{W_1} \times 100 = \frac{2/3 - 4/\pi^2}{2/3} \times 100 = 39.2\%$$

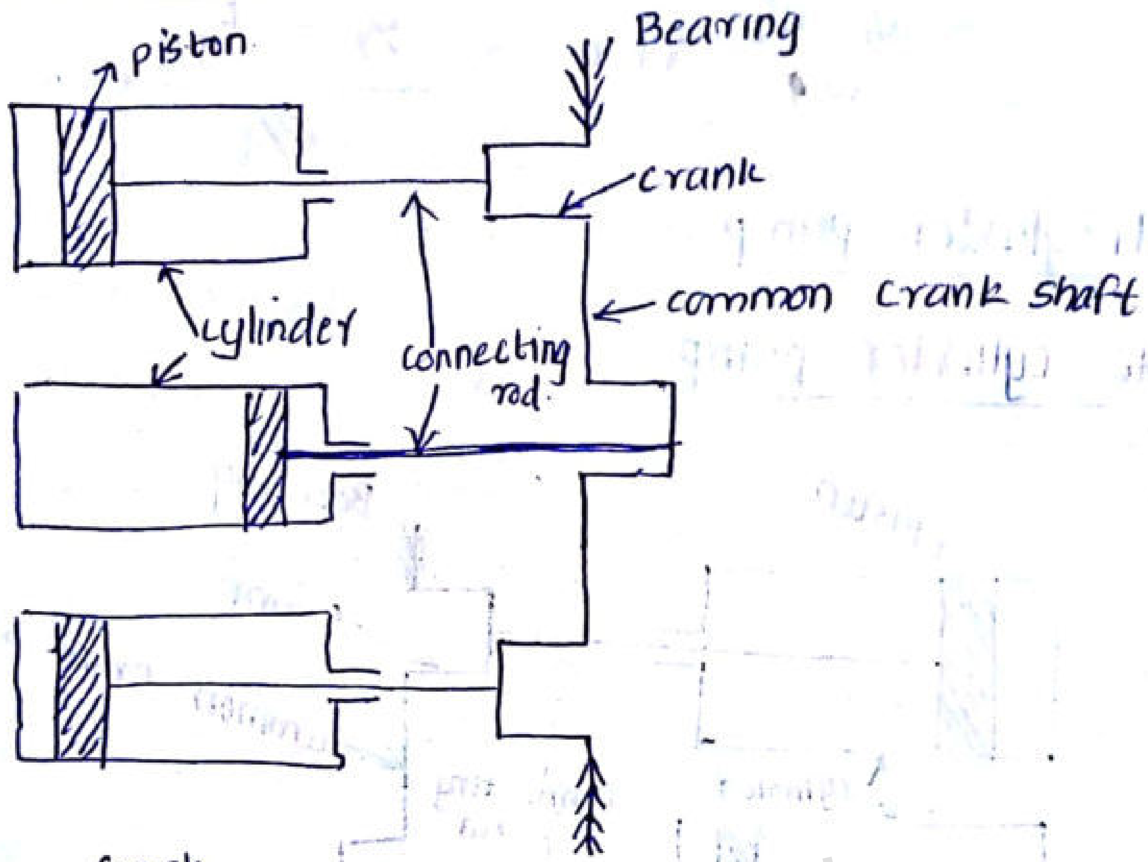
Multicylinder pump.

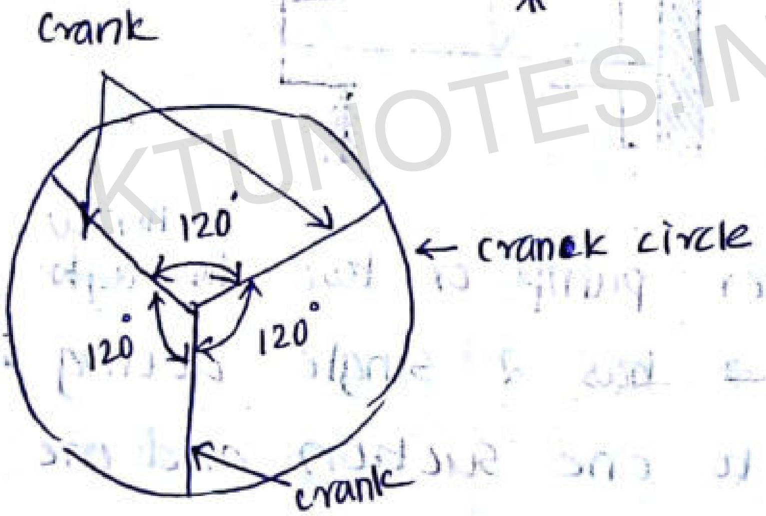
Double cylinder pump.



A double cylinder pump or two ~~through~~ throw pump is one which ~~...~~ has 2 single acting cylinder each equipped to one suction and one delivery pipe with appropriate valves. Two cylinders are connected to a common crank shaft with 2 cranks set at 180°.

Three cylinder pump





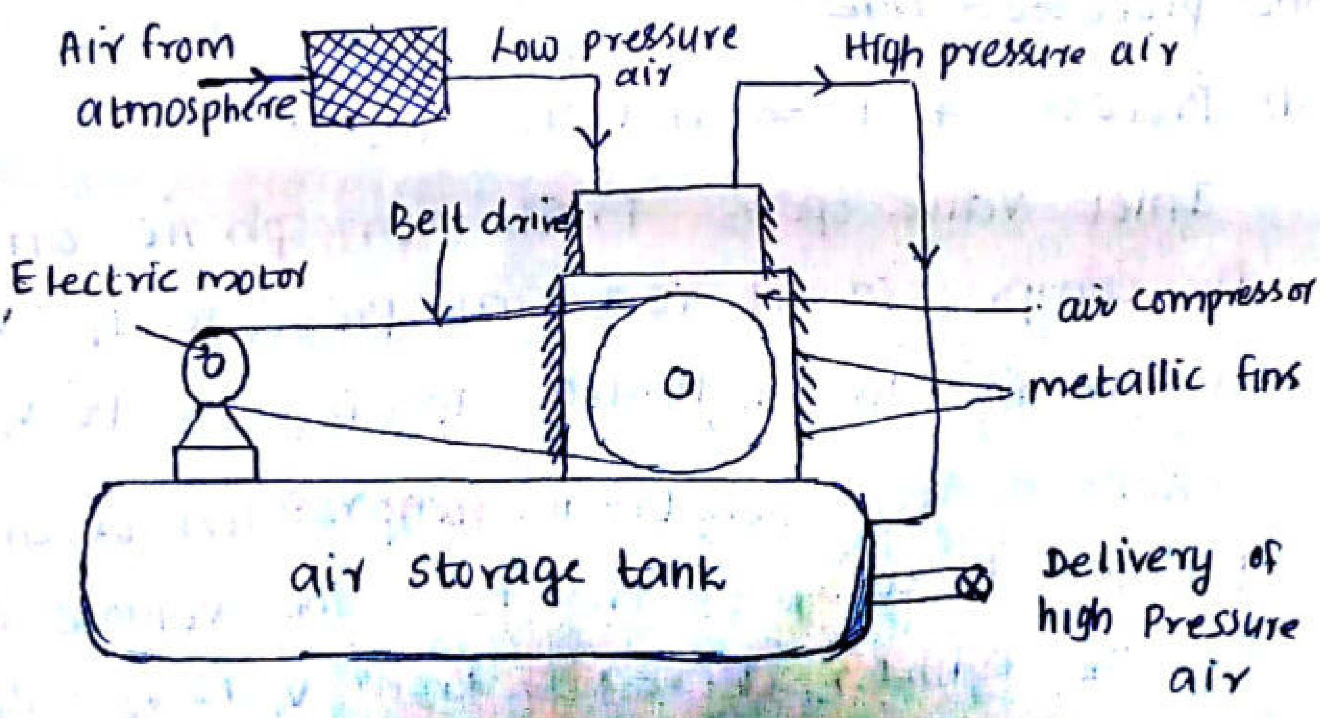
A three cylinder pump or 3 throw pump is one in which there are 3 single acting cylinders, connected to a common crank shaft with 3 cranks set at 120° to each other. The advantage of multicylinder reciprocating pump is that it gives continous flow or large discharge

MODULE - 5 AIR COMPRESSOR

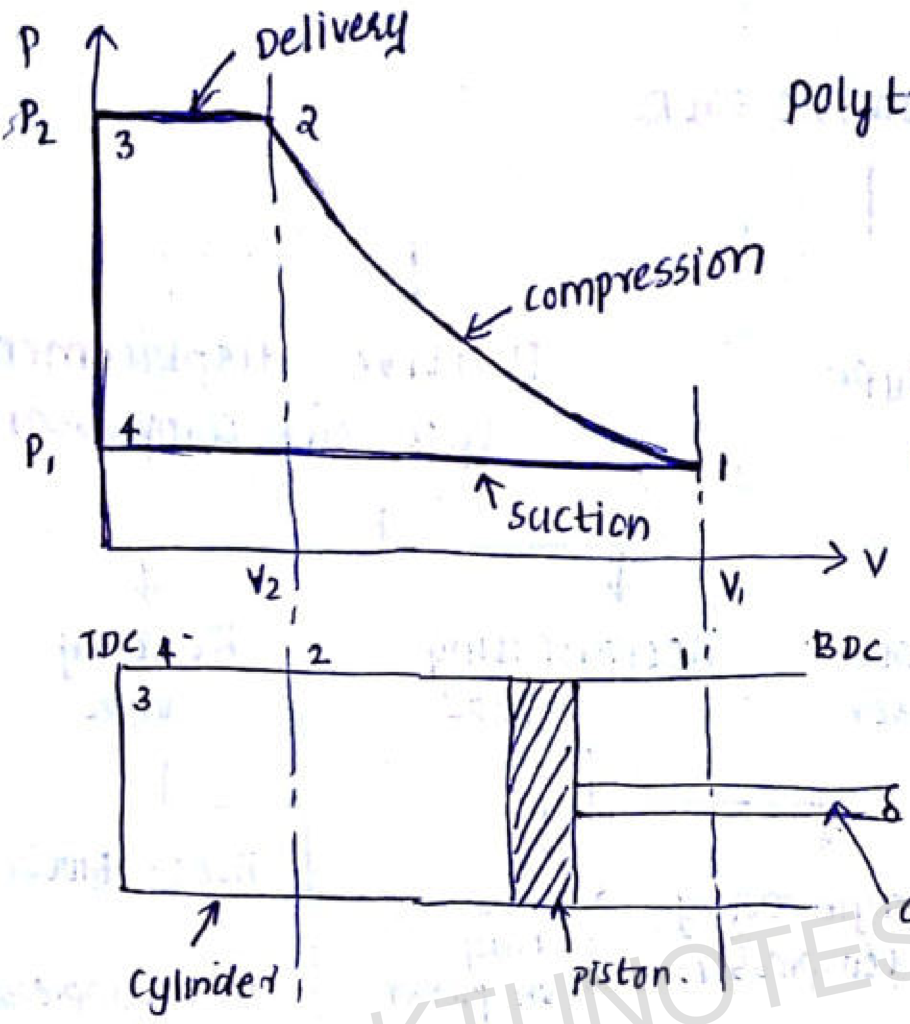
AIR COMPRESSOR

- Dynamic action type air compressor
  - centrifugal air compressor
  - axial flow compressor
- Positive displacement type air compressor
  - Reciprocating type
    - Single acting compressor
    - Double acting compressor
  - Rotary type
    - Roots Blower
    - vane compressor
    - Screw compressor (Lysholm compressor)

Reciprocating air compressor



Working principle



Polytropic compression $pv^n = c$

The figure shows the p-v diagram for a reciprocating compressor without clearance volume for an stg. The processes are

(1) Process 4-1 ⇒ suction stroke

Inlet valve opens. Fresh atmospheric air enters the compressor at constant pressure P_1. volume of air in the cylinder increases to V_1

(2) Process 1-2 ⇒ Polytropic compression of air from pressure P_1 to pressure P_2. The volume of air in the cylinder decreases from V_1 to V_2. the temp.

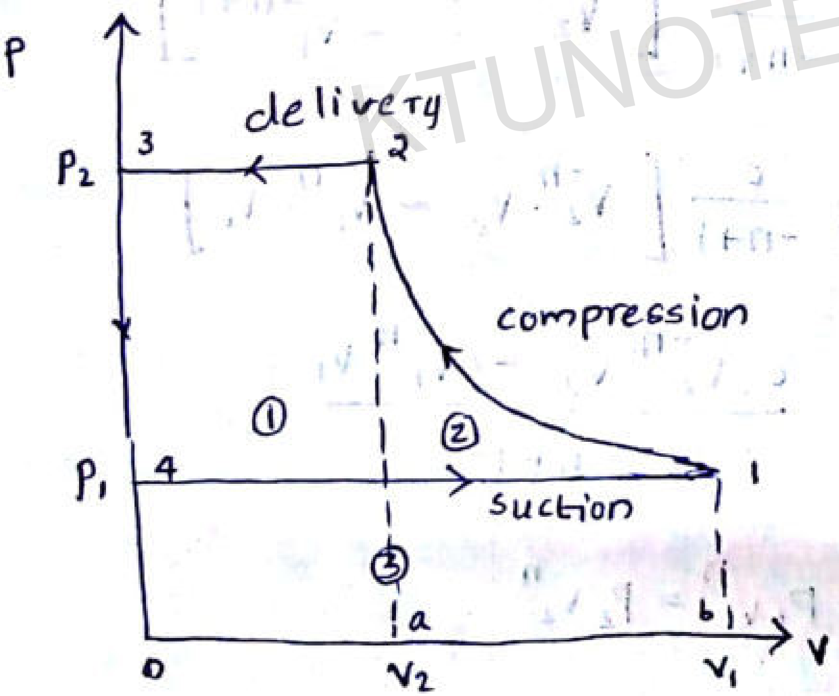
of air increases from T_1 to T_2. At point 2 the delivery valve opens.

(3) Process 2-3 ⇒ Discharge of compressed air through delivery valve at const. Pressure P_2 takes place. Volume of air in the cylinder decreases from V_2 to zero.

(4) Process 3-4 ⇒ No air in the cylinder, and position of piston to start suction stroke

<u>Equation for work input for a single stage compressor</u> (without clearance volume)

~~Net workdone/cycle, W = ∫Pdv~~



Net work done /cycle, W = Area under the curve (4-1-2-3-4)

= Area (3-0-a-2-3) + Area(2-a-b-1-2)

- Area (1-4-0-a-b-1)

$$= P_2 V_2 + \left(-\int_1^2 P\,dv\right) - P_1 V_1 \quad \text{—①}$$

For polytropic process,

$$PV^n = C$$

$$P = \frac{C}{V^n} = CV^{-n}$$

$$\int_1^2 P\,dv = \int_1^2 CV^{-n}\,dv = C\left[\frac{V^{-n+1}}{-n+1}\right]_1^2$$

$$= \frac{C}{-n+1}\left[V_2^{-n+1} - V_1^{-n+1}\right]$$

$$= \frac{C}{-n+1}\left[V_2^{-n} \cdot V_2 - V_1^{-n} \cdot V_1\right]$$

$$= \frac{C V_2^{-n} \cdot V_2 - C \cdot V_1^{-n} V_1}{-n+1}$$

$$\left.\begin{array}{l} PV^n = C \\ \Rightarrow P_1 V_1^n = P_2 V_2^n \end{array}\right\} \; C = P_1 V_1^n = P_2 V_2^n.$$

$$\int_1^2 P\,dv = \frac{P_2 \cancel{V_2^n} \cdot \cancel{V_2^{-n}} \cdot V_2 - P_1 \cancel{V_1^n} \cdot \cancel{V_1^{-n}} \cdot V_1}{-n+1}$$

$$= \frac{P_2 V_2 - P_1 V_1}{-n+1}$$

$$-\int_1^2 P\,dv = \frac{P_2 V_2 - P_1 V_1}{n-1}$$ (multiplying negative on both sides)

substitute in ①

$$W = P_2 V_2 + \frac{P_2 V_2 - P_1 V_1}{n-1} - P_1 V_1$$

$$= P_2 V_2 - P_1 V_1 \left[\frac{1}{n-1} + 1\right]$$

$$= P_2 V_2 - P_1 V_1 \left[\frac{\cancel{1}+n-\cancel{1}}{n-1}\right]$$

$$\boxed{W = \frac{n}{n-1}\left[P_2 V_2 - P_1 V_1\right]} \quad \text{—②} \qquad J/\text{cycle}$$

where, n = polytropic index

P_1 = intake pressure of air

P_2 = Final pressure of air

V_1 = Initial volume

V_2 = Final volume

Modified forms of the above equations.

From eq ②

$$W = \frac{n}{n-1}(P_1 V_1)\left[\frac{P_2 V_2}{P_1 V_1} - 1\right]$$

For polytropic process $PV^n = c$

ie, $P_1 V_1^n = P_2 V_2^n = c$

$$\frac{P_1}{P_2} = \left(\frac{V_2}{V_1}\right)^n \text{ or } \frac{V_2}{V_1} = \left(\frac{P_1}{P_2}\right)^{1/n} = \left(\frac{P_2}{P_1}\right)^{-1/n}$$

$$W = \frac{n}{n-1} P_1 V_1 \left[\frac{P_2}{P_1} \left(\frac{P_2}{P_1}\right)^{-1/n} - 1 \right]$$

$$= \frac{n}{n-1} P_1 V_1 \left[\left(\frac{P_2}{P_1}\right)^{1-1/n} - 1 \right]$$

$$\boxed{W = \frac{n}{n-1} (P_1 V_1) \left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1 \right]}$$

unit: J/cycle

$\frac{P_2}{P_1}$ = Pressure ratio of compressor

$$W = \frac{n}{n-1} m R T_1 \left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1 \right]$$

using characteristic gas equation. ; $P_1 V_1 = mRT_1$

where, m = mass ratio of flow of air n kg/min

R = gas constant of air = 287 J/kgk

T = intake temp of air in kelvin

Efficiencies of Reciprocating Compressors

1) effective pressure, $P_m = \dfrac{\text{work done required per cycle}}{\text{swept volume of cylinder}}$

Swept volume or stroke volume or compressor displacement volume ; $V_1 = \frac{\pi}{4} D^2 \times L$

where, D = Diameter of cylinder or piston
L = stroke length.

2) Indicated power (IP) = Indicated workdone per cycle x No. of cycles per unit time

$= \text{Indicated w/d per cycle} \times \frac{N}{60}$

$= P_m \times L \times A \times \frac{N}{60} \times n$

where ; n = no of suction stroke per revolution of the crank shaft

n=1 ; For single acting compressor
n=2, For double acting compressor

N = Speed of compressor
A = Area of the cylinder or piston.

3) Brake power (BP) or shaft power = Indicated Power (IP) + Friction power (FP)

4) Mechanical efficiency, η_{mech} = $\dfrac{\text{Indicated power required}}{\text{BP or Shaft Power required}}$

5) Adiabactic efficiency, $\eta_{adiabatic}$ = $\dfrac{\text{adiabatic work i/p to the compressor}}{\text{actual work i/p to the compressor}}$

· Work input, $W_{adiabatic} = \dfrac{\gamma}{\gamma-1} P_1 V_1 \left[\left(\dfrac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right]$

For adiabatic process $PV^{\gamma} = C$

where, γ = adiabatic index

6) Isothermal efficiency, $\eta_{Isothermal}$ = $\dfrac{\text{Isothermal work i/p to the compressor}}{\text{Actual work i/p to the compressor}}$

Work input, $W_{isothermal} = P_1 V_1 \log_e \left[\dfrac{P_2}{P_1} \right]$

7) Volumetric efficiency, η_{vol} = $\dfrac{\text{Actual volume of air intake per cycle during suction stroke}}{\text{Theoretical volume of air could fill the swept volume}}$

8) Pressure ratio, $= \dfrac{\text{absolute delivery pressure of air}}{\text{absolute suction pressure of air}}$

$$= \frac{P_2}{P_1}$$

9) volume flow rate of air, $Q = \left[\frac{\pi}{4} D^2\right] L \frac{N}{60} = \frac{ALN}{60}$
(For single acting compressor)

volume flow rate of air ; $Q = 2\left[\frac{\pi}{4} D^2\right] L \cdot \frac{N}{60} = \frac{2ALN}{60}$
(for double acting compressor)

10) Piston Speed $= 2LN$

where N = speed of compressor in rpm

Q) A single acting single cylinder reciprocating air compressor has a cylinder diameter 200mm and a stroke of 300mm. air enters the cylinder at 1 bar, 27°C It is the compressed polytropically to 8 bar accordingly to the law $PV^{1.3} = \text{const}$. If the speed of the compressor is 250 rpm. calculate

(1) the mass of air compressed per minute

(2) the power required in kW for driving the compressor, If $\eta_{mech} = 80\%$. neglect clearance volume

given

$D = 200mm = 0.2m$

$L = 300mm = 0.3 m.$

$P_1 = 1 \text{ bar} = 1\times10^5 N/m^2$

$T_1 = 27°c = 27+273 = 300 K$

$P_2 = 8 \text{ bar} = 8\times10^5 N/m^2$

Polytropic index, $n = 1.3$

$N = 250$ rpm.

$\eta_{mech} = 80\%$

① mass flow rate

we know characteristic gas equation, $P_1V_1 = mRT_1$

Swept volume, $V_1 = \frac{\pi}{4} \cdot D^2 \times L$

$= \frac{\pi}{4} \times 0.2^2 \times 0.3 = 9.424\times10^{-3} m^3$

mass flow rate, $\dot{m} = \frac{P_1V_1}{RT_1}$

$= \frac{1\times10^5 \times 9.424\times10^{-3}}{287\times300}$

$= 0.01094$ kg/cycle

$$= 0.01094 \times N$$

10.424

$$= 0.01094 \times 250$$

$$= 2.736 \text{ kg/min}$$

② Power required in kW (Shaft power)

$$\eta_{mech} = \frac{\text{Indicated power}}{\text{Shaft power}}$$

$$\text{Indicated power} = \text{Indicated work i/p} \times \frac{N}{60}$$

$$= \frac{n}{n-1} P_1 V_1 \left[\left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} - 1 \right] \times \frac{N}{60}$$

$$= \frac{1.3}{1.3-1} (1 \times 10^5 \times 9.424 \times 10^{-3}) \times \left[\left[\frac{8 \times 10^5}{1 \times 10^5} \right]^{\frac{1.3-1}{1.3}} - 1 \right] \times \frac{250}{60}$$

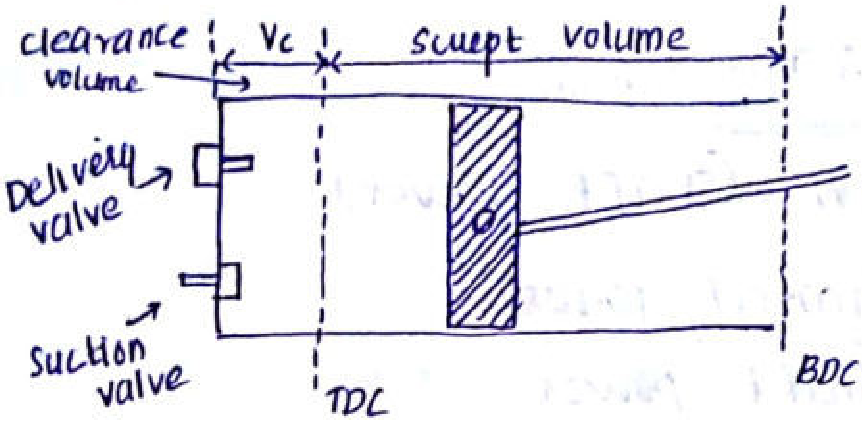
$$= 10.479 \text{ kW}$$

$$\text{Shaft power} = \frac{\text{Indicated power}}{\eta_{mech}}$$

$$= \frac{10.479}{0.8}$$

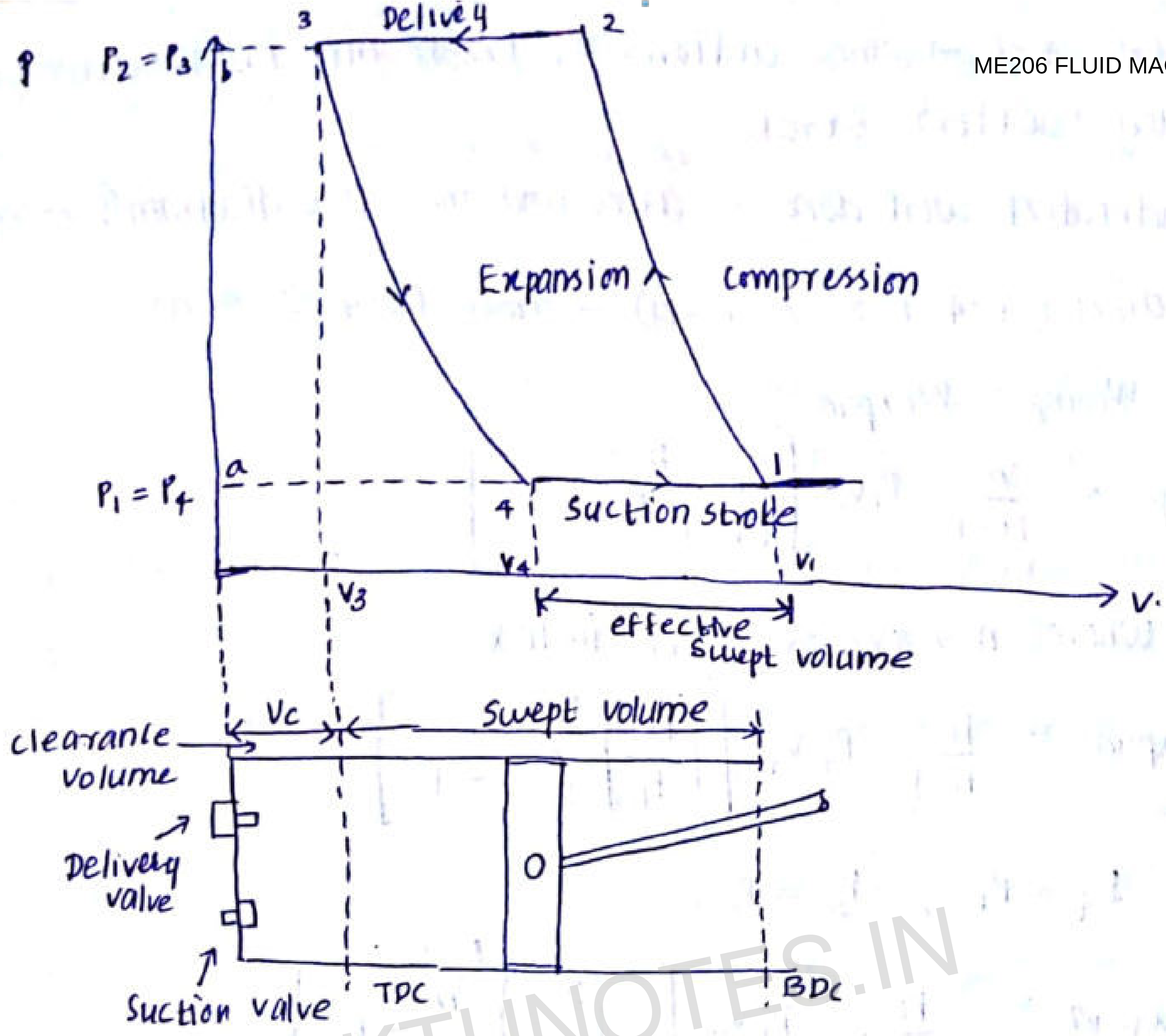
$$= 13.09 \text{ kW}$$

Effect of clearance volume



The clearance volume is the space provided b/w the top dead centre, position of the cylinder and cylinder head. It is provided to prevent the piston from hitting the cylinder head at the end of compression stroke. It also provide ~~the space~~ the space for accomodating the valves actuating mechanism inside the cylinder. suction and delivery valves are located in the clearance volume. The actual volume of air taken to the compressor cylinder per cycle is reduced due to the clearance volume and thus the volumetric efficiency decreases

* **Equation for work input to compressor with considering clearance volume**



process 1-2 ⇒ air is compressed during compression stroke

at stage 2 = delivery valve open

process 2-3 ⇒ air is delivered to the storage tank

at stage-3 = suction stroke starts

process 3-4 ⇒ The compressed air remaining in the clearance volume expands during suction stroke

at stage 4 = pressure drops to P_1 and fresh air from atmosphere starts to enter the cylinder process 4-1 → The suction of fresh air from atmosphere during suction stroke

The indicated work done = area und the P-V diagram (1-2-3-4)

W = area (a-4-1-2-3-b-a) − area (a-4-3-b-a)

$W = W_{comp} - W_{expan}$

$$W_{comp} = \frac{n}{n-1} P_1 V_1 \left[\left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} - 1 \right]$$

where n → compression index

$$W_{expan} = \frac{n}{n-1} P_4 V_4 \left[\left(\frac{P_3}{P_4} \right)^{\frac{n-1}{n}} - 1 \right]$$

$P_4 = P_1$, $P_3 = P_2$

$$W_{expan} = \frac{n}{n-1} P_1 V_4 \left[\left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} - 1 \right]$$

where n → expansion index

$W = W_{comp} - W_{expan}$

$$= \frac{n}{n-1} P_1 V_1 \left[\left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} - 1 \right] - \frac{n}{n-1} P_1 V_4 \left[\left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} - 1 \right]$$

n → compression index

n → expansion index

V_1 → Total volume of cylinder $= V_s + V_c$

on p-v diagram

clearance volume, ~~$V_c = V_3$~~ $V_c - V_3$

swept volume, $V_s = V_1 - V_3$

$= V_1 - V_c$

Q) An ideal single stage single acting reciprocating air compressor has a displacement volume of 14 l and a clearance volume of 0.7 l. It receives the air at a pressure of 1 bar and delivers at a pressure of 7 bar. The compression is polytropic with an index of 1.3 and the re-expansion is isentropic with an index of 1.4. calculate the net indicated work of a cycle

given

Displacement volume = swept volume, $V_s = 14\,l = 14 \times 10^{-3}\,m^3$

clearance volume, $V_c = 0.7\,l = 0.7 \times 10^{-3}\,m^3$

suction pressure, $P_1 = 1$ bar $= 1 \times 10^5\,N/m^2$

Delivery pressure, $P_2 = 7$ bar $= 7 \times 10^5\,N/m^2$

compression index, $n = 1.3$.

Expansion index, $n = 1.4$.

Total volume, $V_1 = V_s + V_c$

$$= 14 \times 10^{-3} + 0.7 \times 10^{-3}$$

$$= \underline{\underline{0.0147 \ m^3}}$$

Taking expansion process, $pv^n = c$

$$P_3 V_3^n = P_4 V_4^n$$

$$P_4 = \frac{P_3 V_3^n}{V_4^n} = P_3 \cdot \left[\frac{V_3}{V_4}\right]^n$$

$$\Rightarrow \frac{P_3}{P_4} = \left[\frac{V_4}{V_3}\right]^n$$

$$\frac{V_4}{V_3} = \left(\frac{P_3}{P_4}\right)^{1/n}$$

We know, $P_3 = P_2$, $P_4 = P_1$

$$V_4 = V_3 \left(\frac{P_2}{P_1}\right)^{1/n}$$

$V_c = V_3 = 0.7 \times 10^{-3} m^3$

$$= 0.7 \times 10^{-3} \times \left[\frac{7 \times 10^5}{1 \times 10^5}\right]^{1/1.4}$$

$$= \underline{\underline{2.810 \times 10^{-3} \ m^3}}$$

$$W = W_{comp} - W_{expan}$$

$$= \frac{n}{n-1} P_1 V_1 \left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1 \right] - \frac{n}{n-1} P_1 V_4 \left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1 \right]$$

$$= \frac{1.3}{1.3-1} (1\times10^5 \times 0.0147) \left[\left(\frac{7\times10^5}{1\times10^5}\right)^{\frac{1.3-1}{1.3}} - 1 \right]$$

$$- \frac{1.4}{1.4-1} (1\times10^5 \times 2.810\times10^{-3}) \left[\left(\frac{7\times10^5}{1\times10^5}\right)^{\frac{1.4-1}{1.4}} - 1 \right]$$

$$= \underline{\underline{2879.35}} \text{ J/cycle}$$

volumetric efficiency

$$\eta_{vol} = \frac{\text{actual volume of air sucked.}}{\text{Theoretical volume of cylinder}}$$

$$= \frac{\text{effective swept volume}}{\text{Swept volume}}$$

$$\boxed{\eta_{vol} = \frac{V_1 - V_4}{V_s}} \quad \text{—(1)}$$

$$\boxed{\eta_{vol} = \frac{V_1 - V_4}{V_1 - V_3}} \quad \text{—(2)}$$

we know, $V_1 = V_S + V_3$

$$\therefore \eta_{vol} = \frac{V_S + V_3 - V_4}{V_S + \cancel{V_3} - \cancel{V_3}}$$

$$= \frac{V_S + V_3 - V_4}{V_S}$$

$$= 1 + \frac{V_3}{V_S} - \frac{V_4}{V_S}$$

$$= 1 + \frac{V_3}{V_S} - \left(\frac{V_4}{V_S}\right) \times \frac{V_3}{V_3}$$

$$= 1 + \frac{V_3}{V_S} - \left(\frac{V_3}{V_S}\right)\left(\frac{V_4}{V_3}\right)$$

But $\frac{V_3}{V_S} = C$; clearance ratio

$$\boxed{\eta_{vol} = 1 + c - c\left(\frac{V_4}{V_3}\right)} \quad \text{--- (3)}$$

For polytropic expansion $PV^n = C$

$$P_3 V_3^n = P_4 V_4^n$$

$$\left(\frac{P_3}{P_4}\right) = \left(\frac{V_4}{V_3}\right)^n$$

$$\frac{V_4}{V_3} = \left(\frac{P_3}{P_4}\right)^{1/n}$$

we know $P_3 = P_2$, $P_4 = P_1$

$$\therefore \frac{V_4}{V_3} = \left(\frac{P_2}{P_1}\right)^{1/n}$$

$$\boxed{\eta_{vol} = 1 + c - c\left(\frac{P_2}{P_1}\right)^{1/n}} \quad \text{--- (4)}$$

Free air Delivery (FAD)

$$FAD = V_1 - V_4$$

{amb = ambient

$$\left[\frac{P_{amb} \cdot V_{amb}}{T_{amb}}\right] = \left[\frac{P_1 \; (V_1 - V_4)}{T_1}\right]_{\text{actual Suction}}$$

$$\boxed{V_{amb}\,(FAD) = \frac{T_{amb}}{P_{amb}} \; \frac{P_1 (V_1 - V_4)}{T_1}}$$

The volume of compressed air delivered corresponding to atmospheric condition is known as free air delivery (FAD) ~~is~~. The volume of compressed air ~~at~~ at stated pressure and temp. of intake air is reduced to atmospheric pressure and temp. It is expressed in m^3/min. Using the relationship between properties of ideal gas, such as pressure, temp and volume

Where

P_{amb} = Pressure of atmospheric air

T_{amb} = Temperature of atmospheric air

V_{amb} = volume of fresh air sucked in to the cylinder during suction stroke at atmospheric condition.

P_1, T_1 = Pressure and temp. of intake air at actual suction conditions

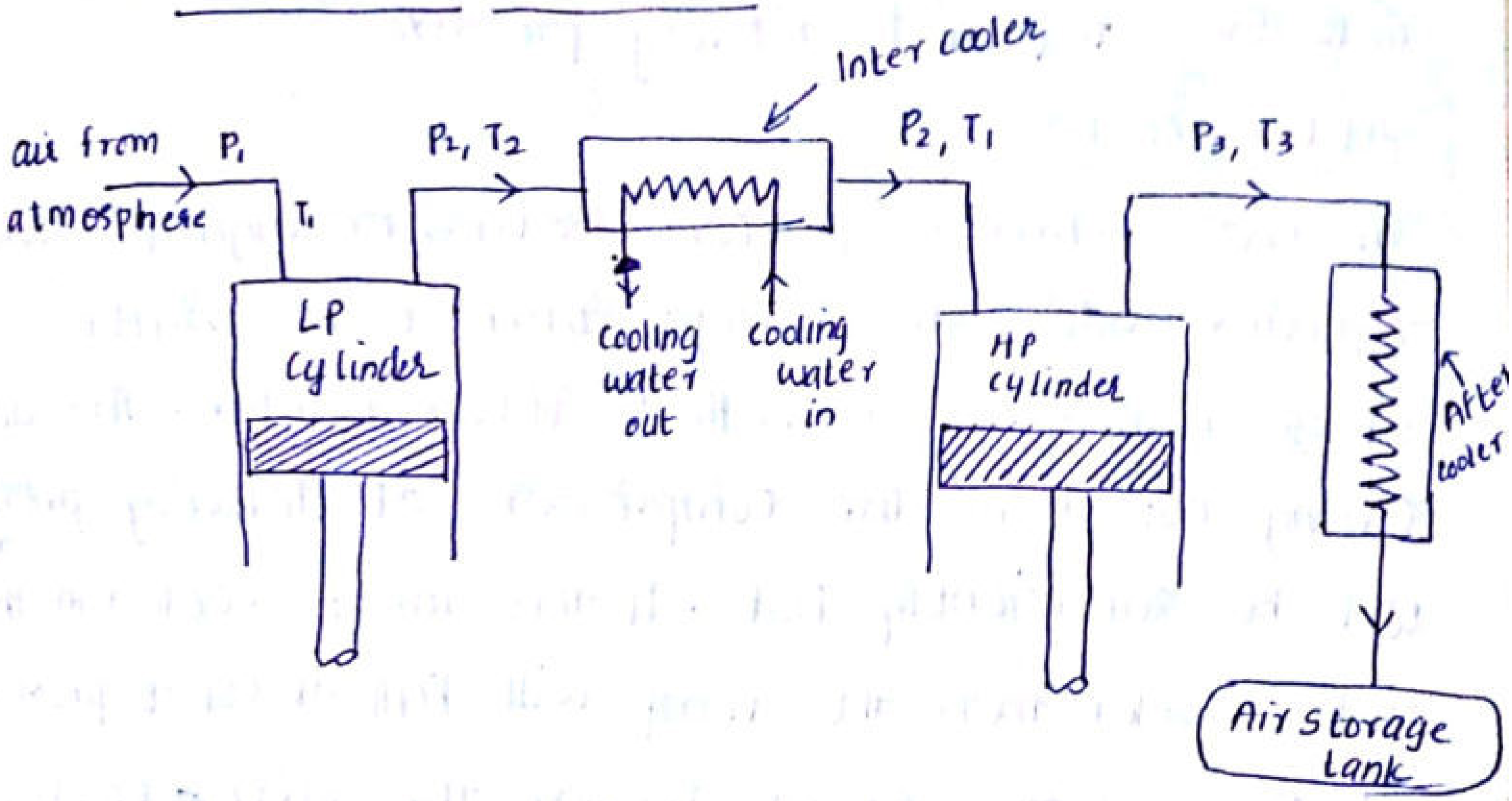
$(V_1 - V_4)$ = effective swept volume

$$\eta_{vol}\,(FAD) = \frac{V_{amb}}{V_s} = \frac{P_1 \times T_{amb}}{P_{amb} \times T_1} \frac{(V_1 - V_4)}{V_1 - V_c}$$

$$\boxed{\eta_{vol} = \frac{P_1 T_{amb}}{P_{amb} \cdot T_1}\left[1 + c - c\left(\frac{P_2}{P_1}\right)^{1/n}\right]}$$

ie, $\frac{V_1 - V_4}{V_1 - V_c}$ = volumetric efficiency at actual suction condition.

Multi stage air compressor



The compression of air in 2 or more cylinders in series with inter cooling between the stages is called multi-stage compression. A multi stage compression is carried out through successive stages till the final delivery pressure and in b/w successive water cooled or air cooled inter coolers are provided.

Inter cooler

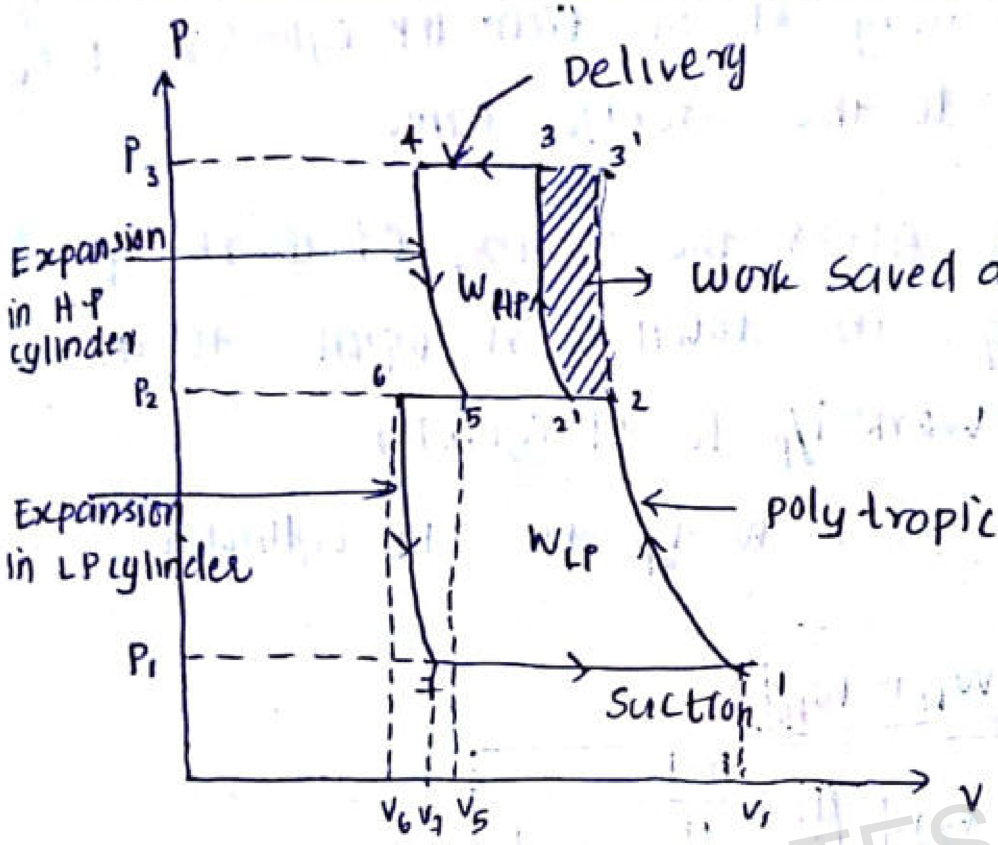
The cooler which is placed between the stages of a multistage compressor is called intercooler. In a 2 stage air compressor, the compressed air at higher temp. from the low pressure cylinder passes in to a inter cooler which is a heat exchanger. The purpose of inter cooler is to reduce the workdone on the air for compressing it in to the required delivery pressure.

After cooler

The cooler which is placed between high pressure cylinder and air storage tank in a multi-stage compressor is called after cooler. The air coming out from the compressor at delivery pressure will be sufficiently hot. If this air is cooled in the after cooler then the temp will fall at const. pressure ∴ The volume of air leaving the after cooler will decrease. So the size of the receiver can be reduced by using after cooler

MODULE-5 (Continu..)..

Workdone in a 2 Stage reciprocating air compressor



Process 7-1 ⇒ Suction in low pressure cylinder and at pressure P_1 and temp T_1

Process 1-2 ⇒ polytropic compression, the air in the low pressure cylinder from pressure P_1 to intermediate pressure P_2

The air in the low pressure cylinder is discharged in to inter cooler where it is cooled at const. pressure P_2 to initial pressure P_1. It is called perfect inter cooler. The line 2 2' represents intercooling.

Process 2'–3 ⇒ polytropic compression from intermediate pressure P_2 to delivery pressure P_3.

Process 3–4 → Delivery of air from HP cylinder at Pressure P_3 to the storage tank

The area 2 2' 3 3' gives the saving of work i/p due to intercooling. The total work input to the compressor W = work i/p to LP cylinder + W i/p to HP cylinder

$$\boxed{W = W_{LP} + W_{HP}}$$

$$\boxed{W_{LP} = \frac{n}{n-1} P_1 (V_1 - V_7) \left[\left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} - 1 \right]}$$

where, n = polytropic index of compression and expansion

$V_1 - V_7$ = effective swept volume in low pressure cylinder

$$\boxed{W_{HP} = \frac{n}{n-1} P_2 (V_{2'} - V_5) \left[\left(\frac{P_3}{P_2} \right)^{\frac{n-1}{n}} - 1 \right]}$$

$V_2' - V_5$ = effective swept volume in HP cylinder

P_3 = delivery pressure

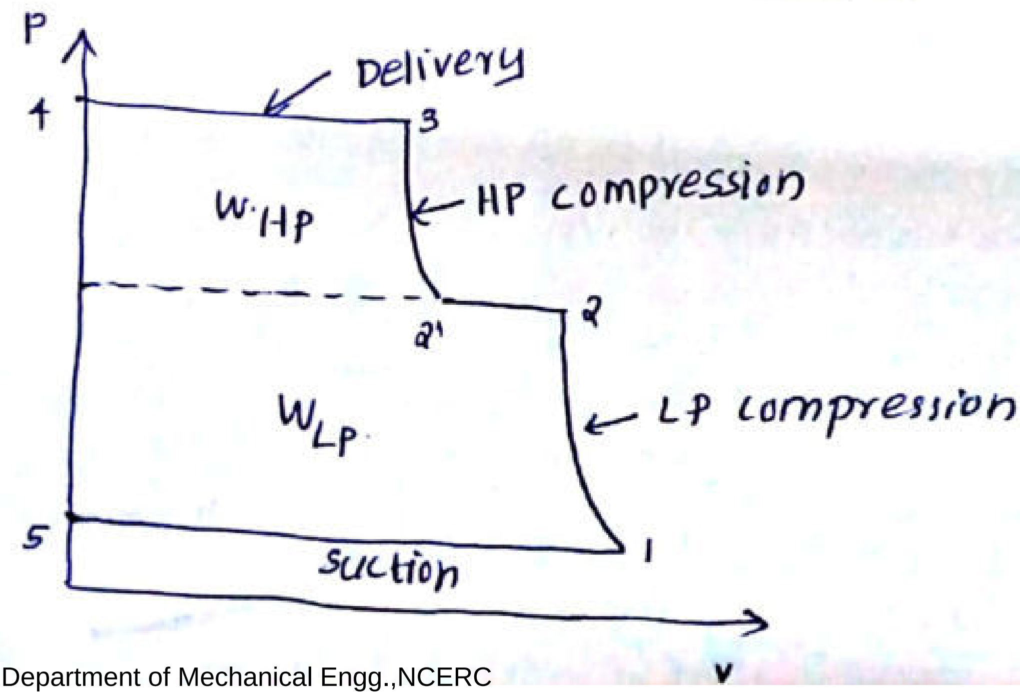
P_2 = intermediate pressure

$$W_{LP} = \frac{n}{n-1} mRT_1 \left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1 \right]$$

$$W_{HP} = \frac{n}{n-1} mRT_1 \left[\left(\frac{P_3}{P_2}\right)^{\frac{n-1}{n}} - 1 \right]$$

Q) Air at 103 kPa and 27°c is drawn in LP cylinder of a two stage air compressor and is isentropically compressed to 700 kPa, air is then cooled at const pressure to 37°c in an inter cooler and is then compressed isentropically to 4 MPa in a high pressure cylinder and delivered at this pressure. Determine the power required to ~~run~~ run the compressor it delivers ~~delivers~~ 30 m³ of air per hour measured at inlet conditions

* Workdone in a two stage reciprocating air compressor without considering clearance volume



$$W_{LP} = \frac{n}{n-1} P_1 V_1 \left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1 \right]$$

$$W_{HP} = \frac{n}{n-1} P_2 V_2' \left[\left(\frac{P_3}{P_2}\right)^{\frac{n-1}{n}} - 1 \right]$$

$W = W_{LP} + W_{HP}$; $n = \gamma = 1.4$

given.

$P_1 = 103\,kPa = 103 \times 10^3\ Pa$

$T_1 = 27^\circ C = 300\ K$

$P_2 = 700\ kPa = 700 \times 10^3\ Pa$

$T_2' = 37^\circ C = 310\ K$

$P_3 = 4\ MPa = 4 \times 10^6\ Pa$

Where $n = \gamma = 1.4$ (For isentropic compression)

V_1 = volume of air delivered

$= 30\ m^3/hr = \frac{30}{60 \times 60}\ m^3/s$

$$W_{L.P} = \frac{n}{n-1} P_1 V_1 \left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1 \right]$$

$$= \frac{1.4}{1.4-1} 103 \times 10^3 \times \frac{30}{60 \times 60} \left[\left(\frac{700 \times 10^3}{103 \times 10^3}\right)^{\frac{1.4-1}{1.4}} - 1 \right]$$

$= 2189.963\ W$

we know, $P_1 V_1 = mRT_1$

$$m = \frac{P_1 V_1}{RT_1}$$

$$= \frac{103 \times 10^3 \times 30}{287 \times 300 \times 60 \times 60}$$

$$= \underline{\underline{9.969 \times 10^{-3}}}$$

$$\underbrace{P_2 V_2' = mRT_2'}\, = mRT_1$$

$$V_2' = \frac{mRT_2'}{P_2} = \frac{9.969 \times 10^{-3} \times 287 \times 310}{700 \times 10^3}$$

$$= \underline{\underline{1.267 \times 10^{-3}}}\ m^3/s$$

$$W_{HP} = \frac{n}{n-1} \times P_2 V_2' \times \left[\left(\frac{P_3}{P_2}\right)^{\frac{n-1}{n}} - 1\right]$$

$$= \frac{1.4}{1.4-1} \times 700 \times 10^3 \times 1.267 \times 10^{-3} \times \left[\left(\frac{4 \times 10^6}{700 \times 10^3}\right)^{\frac{1.4-1}{1.4}} - 1\right]$$

$$= \underline{\underline{2003.4578\ W}}$$

$$W = W_{LP} + W_{HP}$$

$$= 2189.963 + 2003.457$$

$$= 4193.42\ W$$

Q) The L.P cylinder of a 2 stage double acting reciprocating air compressor running at 150 rpm has a 60cm diameter and 80cm stroke. It draws air at a pressure of 1 bar and 25°C and compresses it adiabatically to a pressure of 3 bar. The air is then delivered to the inter cooler where it is cooled at constant pressure to 35°C and is then further compressed polytropically of index $n = 1.3$ to 10 bar in H.P cylinder. Determine the power required to drive the compressor. The mechanical efficiency of the compressor is 85% and motor efficiency is 80%.

given

$N = 150$ rpm.

$D = 60cm = 0.6\ m$

$L = 80cm = 0.8\ m$

$P_1 = 1\ bar = 1 \times 10^5\ N/m^2$

$T_1 = 25°C = 298\ K$

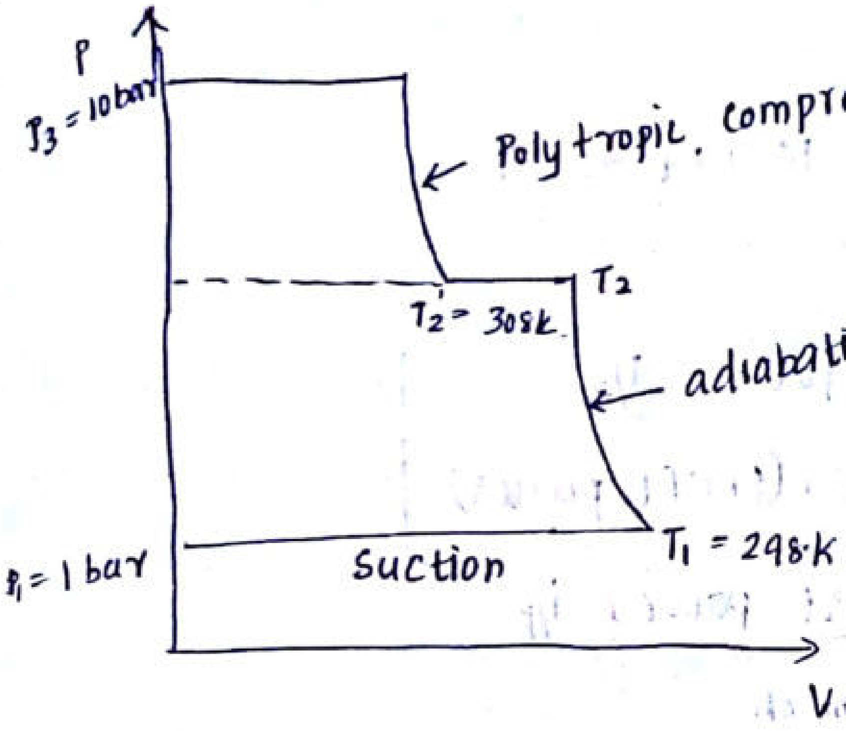
$P_2 = 3\ bar = 3 \times 10^5\ N/m^2$

$T_2' = 35°C = 308\ K$

$P_3 = 10\ bar = 10 \times 10^5\ N/m^2$

$\eta_{mech} = 85\% = 0.85$

$\eta_{motor} = 80\% = 0.8$



$$W_{L.P} = \frac{n}{n-1} P_1 V_1 \left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1 \right]$$

$$= \frac{1.4}{1.4-1} \, 1 \times 10^5 \times 1.1309 \left[\left(\frac{3}{1}\right)^{\frac{1.4-1}{1.4}} - 1 \right]$$

$$= 145961.5408 \text{ W}$$

$$P_1 V_1 = m R T_1$$

$$m = \frac{P_1 V_1}{R T_1} = \frac{1 \times 10^5 \times 1.1309}{287 \times 298} = 1.322$$

$$W_{HP} = \frac{n}{n-1} \times m R T_2' \times \left[\left(\frac{P_3}{P_2}\right)^{\frac{n-1}{n}} - 1 \right]$$

$$= \frac{1.3}{1.3-1} \times 1.322 \times 287 \times 308 \times \left[\left(\frac{10}{3}\right)^{\frac{1.3-1}{1.3}} - 1 \right]$$

$$= 162184.3337 \text{ W}$$

Volume of air sucked

$$V_1 = \frac{\pi}{4} d^2 \times L \times 2$$

$$= \frac{\pi}{4} \times 0.6^2 \times 0.8 \times 2$$

$$= 0.4523 \, m^3/\text{cycle}$$

$$V_1 = 0.4523 \, m^3/\text{cycle}$$

$$= 0.4523 \times \frac{N}{60}$$

$$= 0.4523 \times \frac{150}{60}$$

$$= 1.1309 \, m^3/\text{sec}$$

$$W = W_{LP} + W_{HP}$$

$$= 145961.5408 + 162184.3337$$

$$= \underline{\underline{308145.87\ W}}$$

$$\boxed{\eta_{mech} = \frac{\text{Indicated power i/p}}{\text{Brake power (Shaft power)}}}$$

$$\text{Shaft power} = \frac{\text{indicated power i/p}}{\eta_{mech}}$$

$$= \frac{308.145}{0.85} = \underline{\underline{362.523\ kW}}$$

$$\boxed{\eta_{motor} = \frac{\text{motor power o/p (Shaft power)}}{\text{motor power i/p}}}$$

$$\text{Motor power i/p} = \frac{\text{shaft power}}{\eta_{motor}}$$

$$= \frac{362.523}{0.8} = \underline{\underline{453.153\ kW}}$$

Motor power i/p = 453.153 kW [illegible]

Shaft power (Brake power) = 362.523 kW

Indicated power i/p = 308.145 kW [illegible]

Q) A single stage double acting compressor has a free air delivery of $15\,m^3/min$ measured at 1.013 bar and 15°C. The pressure and temp. in the cylinder during suction are 0.95 bar and 32°C. The delivery pressure is 7 bar and index of compression n = 1.3. The clearance volume is 5% of swept volume calculate indicated power required and volumetric efficiency.

given

$FAD = V_{amb} = 15\,m^3/min$

$P_{amb} = 1.013\text{ bar} = 1.013 \times 10^5\text{ Pa}$

$T_{amb} = 15^\circ C = 288\,K$

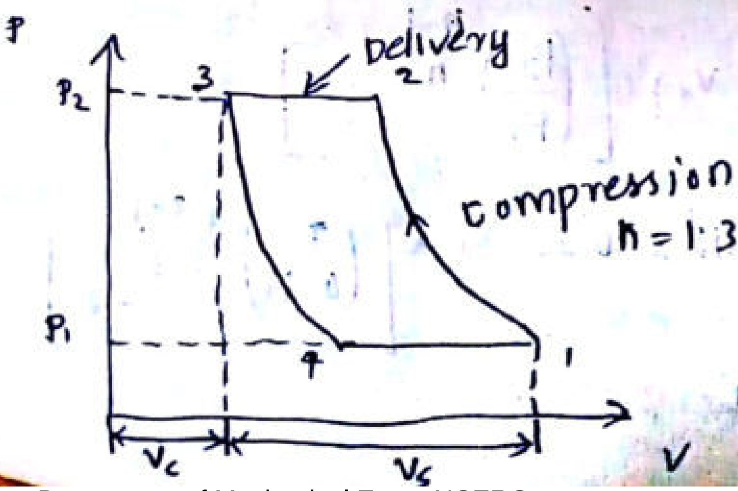
$P_1 = 0.95\text{ bar} = 0.95 \times 10^5\,Pa$

$T_1 = 32^\circ C = 305\,K$

$P_2 = 7\text{ bar} = 7 \times 10^5\text{ Pa}$

$n = 1.3$

$V_c = 0.05\,V_s$



$$\boxed{\eta'_{vol}(FAD) = \frac{P_1}{T_1} \times \frac{T_{amb}}{P_{amb}}\left[1 + c - c\left(\frac{P_2}{P_1}\right)^{1/n}\right]}$$

$$= \frac{0.95\times10^5}{305} \times \frac{288}{1.013\times10^5} \times \left[1 + 0.05 - 0.05\times\left[\frac{7}{0.95}\right]^{1/1.3}\right]$$

c = clearance ratio $= \frac{V_c}{V_s} = 0.05$

$$= 0.7240 = 72.4\%$$

From FAD equation

~~Indicated power input~~

$$\boxed{\frac{P_{amb} \cdot V_{amb}}{T_{amb}} = \left[\frac{P_1}{T_1}[V_1 - V_4]\right]_{actual}}$$

$$V_1 - V_4 = \frac{P_{amb} \cdot V_{amb}}{T_{amb}} \times \frac{T_1}{P_1}$$

$$= \frac{1.013\times10^5 \times 15\times 1/60}{288} \times \frac{305}{0.95\times10^5}$$

$$= 0.28231$$

Indicated power input,

$$W = \frac{n}{n-1} P_1 (V_1 - V_4)\left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1\right]$$

$$= \frac{1.3}{1.3-1}\, 0.95\times10^5 \times 0.28231 \times \left[\left(\frac{7}{0.95}\right)^{\frac{1.3-1}{1.3}} - 1\right]$$

$$= 68044.17 \text{ W}$$

Q) In a single acting 2 stage reciprocating air compressor 5 kg of air per minute is compressed from 1·013 bar and 20°c through a pressure ratio of 10 ~~then~~. both stages have same pressure ratio and perfect intercooler both stages, the law of compression and law of expansion is $Pv^{1.3}$ = const. calculate

(1) Indicated power

(2) The cylinder swept volume required

assume that the clearance volume of both stages are 5% of their respective swept volumes and compressor runs at 325 rpm and R = 287 J/kgK

given

$m = 5\ kg/min = \frac{5}{60}\ kg/s$

$P_1 = 1.013 \times 10^5\ Pa$

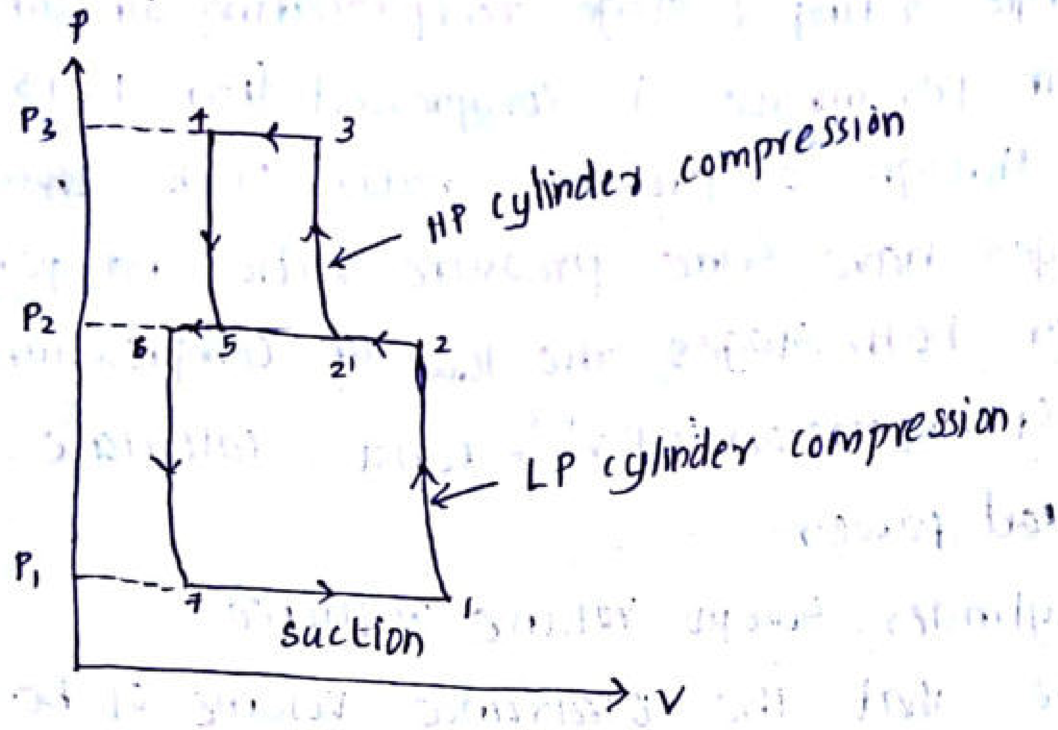
$T_1 = 20°C = 293\ K$

$n = 1.3$

$V_c = 5\%\ V_s = 0.05\ V_s$

Pressure ratio $= \frac{P_3}{P_1} = 10$

$N = 325\ rpm$



$$W_{LP} = \frac{n}{n-1} P_1 (V_1 - V_7) \left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1 \right]$$

$$= \frac{n}{n-1} m R T_1 \left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1 \right]$$

For perfect intercooling, pressure ratio's are same

$$\left(\frac{P_2}{P_1}\right)_{\text{first stage}} = \left(\frac{P_3}{P_2}\right)_{\text{second stage}}$$

$$P_2^2 = P_1 \times P_3$$

$$P_2 = \sqrt{P_1 \times P_3}$$

$$= \sqrt{1.013 \times 10^5 \times 10 \times 1.013 \times 10^5}$$

$$= \underline{320338.727 \text{ Pa}}$$

But $\frac{P_3}{P_1} = 10$

$P_3 = 10 P_1$

$$W_{LP} = \frac{n}{n-1} mRT_1 \left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1 \right]$$

$$= \frac{1.3}{1.3-1} \times \frac{5}{60} \times 287 \times 293 \left[\left[\frac{320338.727}{1.013 \times 10^5} \right]^{\frac{1.3-1}{1.3}} - 1 \right]$$

$$= \underline{\underline{9241.082 \text{ W}}}$$

$$W_{HP} = \frac{n}{n-1} P_1 (V_2' - V_5) \left[\left(\frac{P_3}{P_2}\right)^{\frac{n-1}{n}} - 1 \right]$$

$$= \frac{n}{n-1} mRT_2' \left[\left(\frac{P_3}{P_2}\right)^{\frac{n-1}{n}} - 1 \right]$$

For perfect intercooling, $T_1 = T_2'$ and pressure

$$W_{HP} = \frac{n}{n-1} mRT_1 \left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1 \right]$$

$$= \frac{1.3}{1.3-1} \times \frac{5}{60} \times 287 \times 293 \left[\left(\frac{320338.727}{1.013 \times 10^5}\right)^{\frac{1.3-1}{1.3}} - 1 \right]$$

$$= \underline{\underline{9241.082 \text{ W}}}$$

$$IP, W = W_{HP} + W_{LP}$$

$$= 2 \times 9241.082$$

$$= \underline{\underline{18482.164 \text{ W}}}$$

$$\boxed{\eta_{vol} = \frac{\text{effective swept volume}}{\text{swept volume}}}$$

$$\eta_{vol_{LP}} = \frac{V_1 - V_7}{\text{Swept volume}}$$

$$\eta_{vol} = 1 + c - c\left(\frac{P_2}{P_1}\right)^{1/n}$$

$$= 1 + 0.05 - 0.05 \times \left[\frac{320338.727}{1.013 \times 10^5}\right]^{1/1.3} \quad \left\{ c = \frac{V_c}{V_s} = 0.05 \right.$$

$$= \underline{\underline{0.9287}}$$

$$P_1 (V_1 - V_7) = m R T_1$$

$$V_1 - V_7 = \frac{m R T_1}{P_1} = \frac{5/60 \times 287 \times 293}{1.013 \times 10^5}$$

$$= \underline{\underline{0.06917}}$$

$$\eta_{vol_{LP}} = \frac{V_1 - V_7}{\text{Swept volume}}$$

$$\Rightarrow \text{Swept volume} = \frac{V_1 - V_7}{\eta_{vol}}$$

$$= \frac{0.06917}{0.9287} = \underline{\underline{0.0744 \ m^3/sec}}$$

MODULE - 6 ROTARY COMPRESSORS

Rotary compressors

Rotary compressors are the machines which are used to supply continous pulsation free compressed air at a comparitively low and medium pressures. The low starting torque of the rotary compressor helps to connect directly with the electric motor. Rotary compressors are compact, well balanced and high speed compressors.

Rotary Compressors
- Positive displacement type
  - Screw compressor
  - Roots Blower
  - Vane compressor
- Dynamic action type
  - centrifugal compressor
  - axial flow compressor

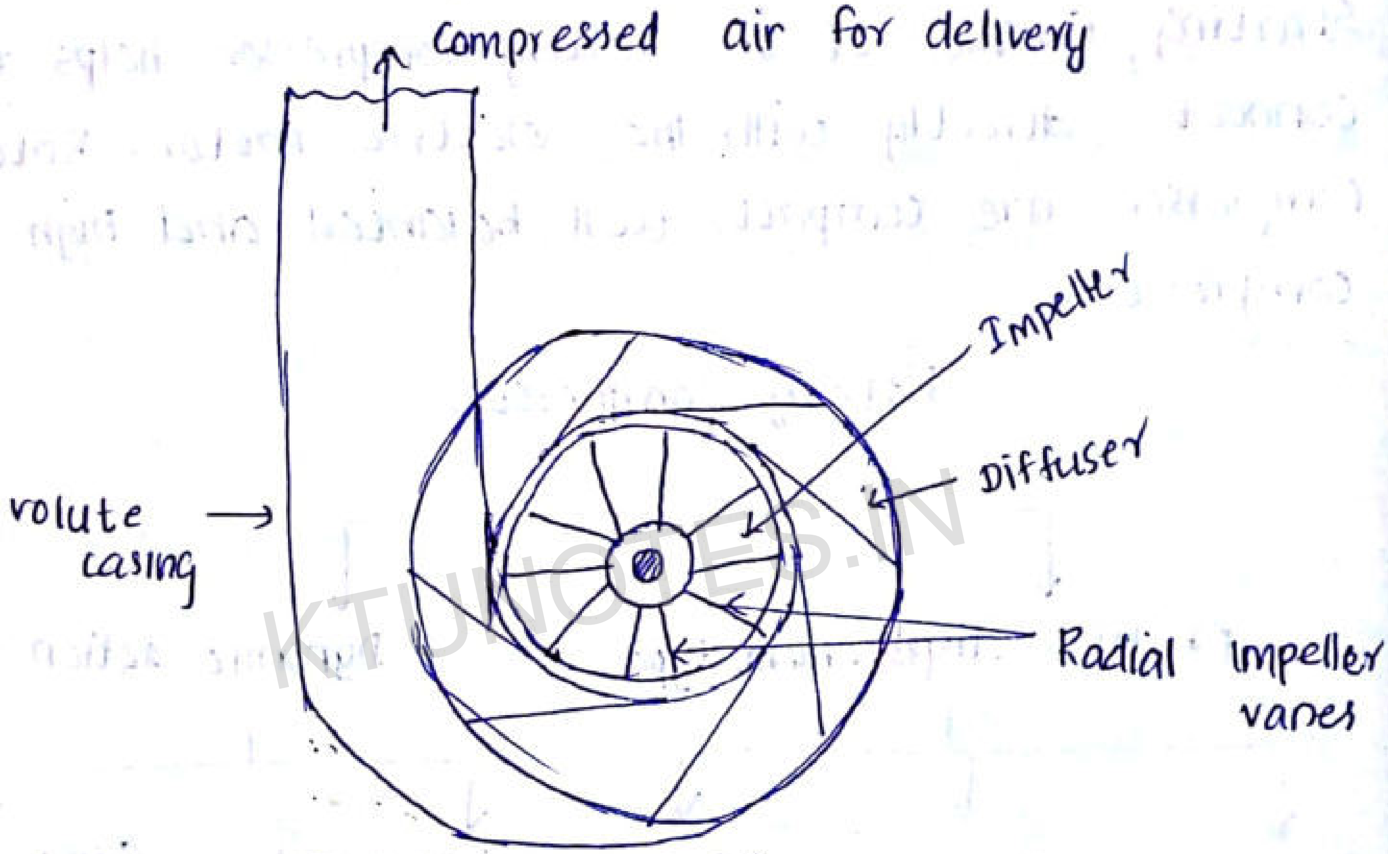
Positive displacement type compressor

In this type, the air is compressed by being trapped in the reduced space formed by means of two sets of engaging surfaces and the pressure of air is increased by squeezing action.

Dynamic action type compressor

In this type air is not trapped in specified boundaries but the air flows continously and pressure is increased due to dynamic action of centrifugal force

centrifugal compressor



The main parts of centrifugal compressors are

(1) A rotating impeller

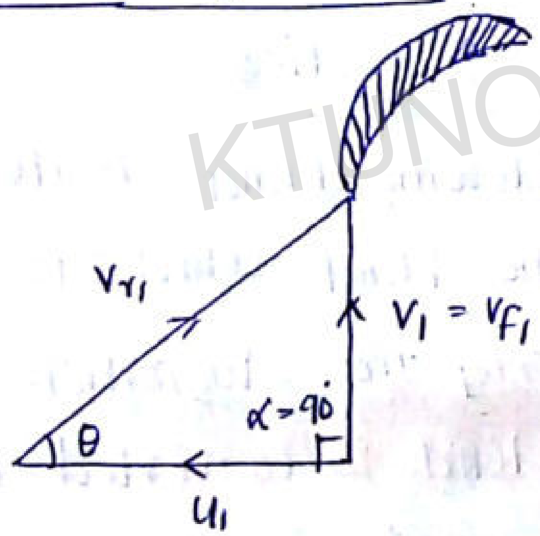
(2) A diffuser

(3) A ~~seal~~ Stationery casing

The rotating impeller is a radial disc on which a series a radially blades are attached. The impeller rotates inside the stationery casing and the centre of the impeller is called the eye of the impeller. The difusser which surrounds the impeller and provides diverging passage for air flow. The main function of the diffuser is to convert high velocity head of the air into static pressure head. The air coming out from the diffuser is collected in the casing and taken out from the outlet of the compressor. The casing of the compressor has volute type and surrounds the impeller.

velocity triangles of centrifugal compressor

Inlet velocity triangle

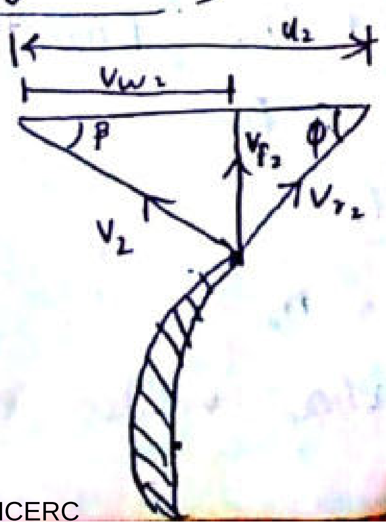


$V_1, V_2 \Rightarrow$ vane velocity

$V_{r_1}, V_{r_2} \Rightarrow$ Jet velocity

$\boxed{V_{w_1} = 0}$

outlet velocity triangle



β = inlet angle of diffuser

Work done by the impeller

Workdone by the impeller/sec

= torque developed x angular velocity

$= T \times \omega$

$= (m\, V_{w_2}\, R_2) \times \omega$

$\boxed{W/sec = m\, V_{w_2}\, u_2}$ $\left\{ \because u_2 = \omega R_2 \right.$

$\boxed{\text{work done/sec/kg of air} = V_{w_2} u_2}$ $\because m = 1\,kg$

ω = angular velocity of impeller

m = mass rate of flow.

Concept of Stagnation Properties

Stagnation state of a flowing fluid is defined as the state attained by the fluid which is brought to rest isentropically. During the stagnation process kinetic energy of the fluid is converted into enthalpy which results in the increase of pressure and temp. of the fluid.

Stagnation pressure = P_0

Stagnation temp. = T_0

Stagnation enthalpy = h_0

P, T, h are corresponding values at static stage.

Static enthalpy, $h = mc_p \cdot \Delta T$

Stagnation enthalpy, $h_0 = h + \frac{V^2}{2} = mc_p \Delta T_0$ —①

$$= mc_p \Delta T + \frac{V^2}{2} \quad —②$$

equating ① and ②

and considering state 1.

$$mc_p \Delta T_{0_1} = mc_p \Delta T_1 + \frac{V_1^2}{2}$$

corresponding the state for m = 1kg

$$c_p T_{0_1} = c_p T_1 + \frac{V_1^2}{2}$$

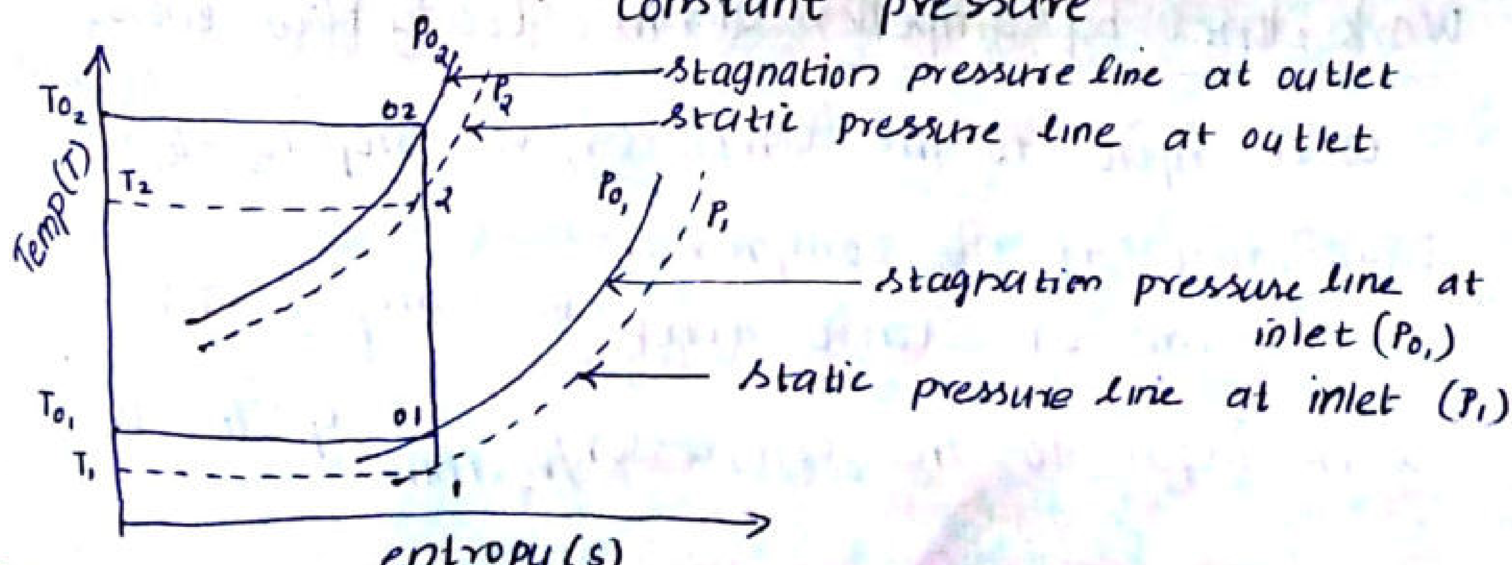
$$\boxed{T_{0_1} = T_1 + \frac{V_1^2}{2c_p}}$$

where T_{0_1} is the stagnation temp at inlet

T_1 = static temp at inlet

V_1 = velocity of fluid at inlet

c_p - Specific heat of air or fluid at constant pressure



From the graph,

Using isentropic relation of perfect gas,

$$\frac{T_{0_1}}{T_1} = \left[\frac{P_{0_1}}{P_1}\right]^{\frac{\gamma-1}{\gamma}}$$

Similarly,

$$\frac{T_{0_2}}{T_2} = \left[\frac{P_{0_2}}{P_2}\right]^{\frac{\gamma-1}{\gamma}}$$

P_{0_1} = stagnation pressure at inlet

P_1 = Static pressure at inlet

P_{0_2} = Stagnation pressure at outlet

P_2 = Static pressure at inlet

γ = isentropic index $= \frac{c_p}{c_v}$

c_p = specific heat at const Pressure

c_v = specific heat at const volume

Work done by Impeller (From steady flow energy equation)

work input to the compressor, $W = mc_p(T_{0_2} - T_{0_1})$

Work input to the compressor in terms of static temp, $W = mc_p(T_2 - T_1)$

work input to the compressor/kg of air $= c_p(T_{0_2} - T_{0_1})$

$$= C_p T_{0_1}\left[\left(\frac{P_{0_2}}{P_{0_1}}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]$$

If velocity of inlet = velocity of outlet

$$V_1 = V_2$$

Then, W/kg of air $= C_p T_1 \left[\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]$

Width of the impeller

we know,

Discharge through impeller, Q = Area of impeller x velocity of flow

$$Q_1 = A_1 \cdot V_{f_1}$$

$$Q_1 = \pi D_1 B_1 V_{f_1}$$

width of the impeller at inlet, $B_1 = \dfrac{Q_1}{\pi D_1 V_{f_1}}$

mass ~~flow~~ rate of flow, $m = \rho_1 Q_1$

$$m = \rho_1 \cdot \pi D_1 B_1 V_{f_1}$$

$$\boxed{B_1 = \frac{m}{\rho_1 \pi D_1 V_{f_1}}}$$

If considering thickness of Blades and number of blades

Area of Flow $= (\pi D_1 - nt) \times B_1$

Discharge, $Q = (\pi D_1 - nt) \times B_1 \times V_{f_1}$

mass rate of flow, $m = \rho_1 \times (\pi D_1 - nt) B_1 \times V_{f_1}$

$$\boxed{B_1 = \frac{m}{\rho_1 (\pi D_1 - nt) V_{f_1}}}$$

where n = no of blades

t = thickness of blades

Degree of Reaction

$$\text{Degree of reaction} = \frac{\text{Static pressure rise in the impeller}}{\text{Total static pressure rise in the impeller}}$$

$$\boxed{\text{Degree of reaction} = 1 - \frac{V_{w_2}}{2u_2}}$$

Q) A centrifugal compressor running at 1500 rpm has internal and external diameters of the impeller as 250mm and 500mm respectively. The blades angles at inlet and outlet are 18° and 40° respectively. the air enters the impeller radially. Determine the work done by the compressor per kg of air and degree of reaction.

given

$N = 1500$ rpm

$D_1 = 250mm = 0.25 m$

$D_2 = 500 mm = 0.5 m$

$\theta = 18°$

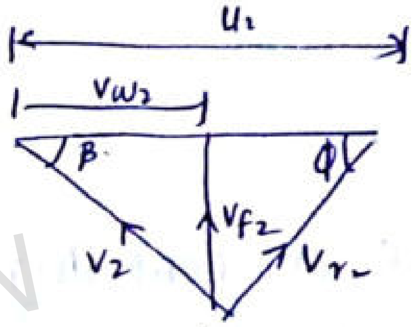
$\phi = 40°$

$\alpha = 90°$ (air enters radially)

$V_{w_1} = 0$

(i) work done by the compressor/kg of air $= V_{w_2} u_2$

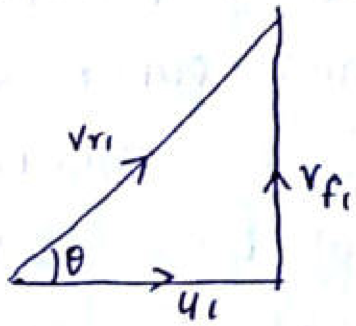
$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.5 \times 1500}{60} = \underline{\underline{39.269}} \text{ m/s}$$



~~tan~~

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.25 \times 1500}{60} = \underline{\underline{19.634}} \text{ m/s}$$

$$\tan\theta = \frac{V_{f_1}}{u_1}$$



$$V_{f_2} = V_{f_1} = u_1 \cdot \tan\theta$$

$$= 19.634 \times \tan 18°$$

$$= \underline{\underline{6.379}} \text{ m/s}$$

$$\tan\phi = \frac{V_{f_2}}{u_2 - V_{w_2}}$$

$$u_2 - V_{w_2} = \frac{V_{f_2}}{\tan\phi}$$

$$V_{w_2} = u_2 - \frac{V_{f_2}}{\tan\phi} = 39.269 - \frac{6.379}{\tan 40}$$

$$= 31.666 \text{ m/s}$$

Work done by the compressor /kg of air $= V_{w_2} \cdot u_2$

$= 31.666 \times 39.269$

$= \underline{\underline{1243.523}}$ N/kg

Degree of reaction $= 1 - \dfrac{V_{w_2}}{2u_2}$

$= 1 - \dfrac{31.666}{2 \times 39.269}$

$= 0.596$

$= \underline{\underline{59.6\%}}$

Q) A centrifugal compressor running at 1500 rpm handles air at 1 bar and 25°c and compresses it to a pressure of 6 bar isentropically. The inner and outer diameter of impeller are 15cm and 30cm respectively. The width of the blade at inlet is 2.75 cm. The blade angles are 18° and 40° at entry and exit. calculate

(1) mass rate of flow of air

(2) Degree of reaction

(3) Power input

(4) Width of the blade at outlet

given

$N = 1500$ rpm

$P_1 = 1$ bar $= 1 \times 10^5$ N/m²

$T_1 = 25°C = 298$ K

$P_2 = 6$ bar $= 6 \times 10^5$ N/m²

$D_1 = 15$ cm $= 0.15$ m

$D_2 = 30$ cm $= 0.3$ m

$B_1 = 2.75$ cm $= 0.0275$ m

$\theta = 18°$

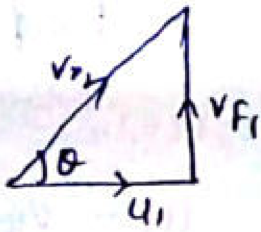
$\phi = 40°$

1. mass rate of flow of air, $m = \rho_1 Q_1$

$$Q_1 = \pi D_1 B_1 V_{f_1}$$

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.15 \times 1500}{60} = 11.780 \text{ m/s}$$

$$\tan\theta = \frac{V_{f_1}}{u_1}$$



$$V_{f_1} = u_1 \tan\theta$$

$$= 11.780 \times \tan 18$$

$$= 3.827 \text{ m/s}$$

$$\therefore Q_1 = \pi D_1 B_1 V_{f_1}$$

$$= \pi \times 0.15 \times 0.0275 \times 3.827$$

$$= 0.0495 \text{ m}^3/\text{s}$$

According to perfect gas equation

$$P_1 = \rho_1 R T_1$$

$R = 287\ J/kgK$

$$\rho_1 = \frac{P_1}{RT_1}$$

$$= \frac{1\times10^5}{287 \times 298}$$

$$= \underline{\underline{1.169}}\ kg/m^3$$

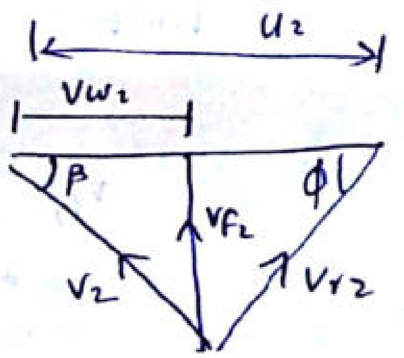
$$m = \rho_1 Q_1 = 1.169 \times 0.0495$$

$$= \underline{\underline{0.0578}}\ kg/s$$

2. Degree of reaction $= 1 - \dfrac{V_{w_2}}{2u_2}$

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.3 \times 1500}{60} = \underline{\underline{23.561}}\ m/s$$

$$\tan\phi = \frac{V_{f_2}}{u_2 - V_{w_2}}$$



$$u_2 - V_{w_2} = \frac{V_{f_2}}{\tan\phi}$$

$$V_{w_2} = u_2 - \frac{V_{f_2}}{\tan\phi}$$

$V_{f_2} = V_{f_1}$

$$= 23.561 - \frac{3.827}{\tan 40} = \underline{\underline{19}}\ m/s$$

$\therefore$ Degree of reaction $= 1 - \dfrac{V_{w_2}}{2u_2}$

$$= 1 - \frac{19}{2 \times 23.561}$$

$$= 0.5967$$

$$= 59.67\,\%$$

③ Power input = work done /sec

$$= m \cdot V_{w_2} \cdot u_2$$

$$= 0.0578 \times 19 \times 23.561$$

$$= 25.874 \text{ W}$$

4. width of impeller outlet

$$B_2 = \frac{m}{\rho_2 \cdot \pi D_2 \cdot V_{f_2}}$$

$$P_2 = \rho_2 R T_2$$

$$\rho_2 = \frac{P_2}{R T_2} \rightarrow \cancel{6 \times 10^5}$$

For getting T_2, using isentropic compression relation.

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

{ For isentropic process $\gamma = 1.4$

$$T_2 = T_1 \left[\frac{P_2}{P_1}\right]^{\frac{\gamma-1}{\gamma}}$$

$$= 298 \times \left[\frac{6}{1}\right]^{\frac{0.4}{1.4}} = 497.216 \text{ K}$$

$$\rho_2 = \frac{P_2}{RT_2}$$

$$= \frac{6 \times 10^5}{287 \times 497.216}$$

$$= \underline{4.204} \ kg/m^3$$

~~RT_2~~

$$B_2 = \frac{m}{\rho_2 \times \pi D_2 \cdot V_{f_2}}$$

$$= \frac{0.0578}{4.204 \times \pi \times 0.3 \times 3.827}$$

$$= 3.811 \times 10^{-3} \ m$$

Slip of rotary compressor

The difference between, the impeller blade velocity at outlet (u_2) and velocity of whirl of air outlet (V_{w_2}) is known as slip

$$\boxed{\text{Slip} = u_2 - V_{w_2}}$$

$u_2 \Rightarrow$ impeller blade or vane velocity at outlet
$V_{w_2} \Rightarrow$ velocity of whirl of air at outlet

IMP

Slip factor (ϕ_s)

The ratio of velocity of whirl of air at outlet to the impeller blade velocity at outlet is known as slip factor.

$$\text{slip factor, } \phi_s = \frac{V_{w_2}}{u_2}$$

Theoretically $u_2 = V_{w_2}$

In actual $u_2 \neq V_{w_2}$

Work factor (ϕ_w)

Work factor or power input factor is defined as the ratio of actual work input by compressor to the idle impeller work input to the air

$$\text{work factor, } \phi_w = \frac{\text{actual work input by the compressor per kg of air}}{\text{Impeller work i/p to air/kg of air}}$$

actual work i/p by the compressor $= m c_p (T_{0_2} - T_{0_1})$

actual work i/p by the compressor/kg of air $= c_p (T_{0_2} - T_{0_1})$

Impeller work i/p to air/kg of air $= V_{w_2} u_2$

$$\boxed{\text{work factor, } \phi_w = \frac{c_p (T_{0_2} - T_{0_1})}{V_{w_2} u_2}}$$

Pressure coefficient (ϕ_p)

Pressure coefficient is defined as the ratio of Isentropic work input by the compressor to the impeller work input to the air

$$\phi_p = \frac{\text{isentropic work input by the compressor/kg of air}}{\text{Impeller work input to air /kg of air}}$$

isentropic work input by the compressor $= mC_p[T_{02}' - T_{01}]$

isentropic work input by the compressor per kg of air $= C_p[T_{02}' - T_{01}]$

Impeller work input to air/kg of air $= V_{w_2} u_2$

$$\boxed{\text{Pressure coefficient, } \phi_p = \frac{C_p[T_{02}' - T_{01}]}{V_{w_2} u_2}}$$

where, T_{01} = Stagnation temp at inlet.

T_{02} = Stagnation temp at outlet

T_{02}' = stagnation temp at outlet in isentropic process

we know,

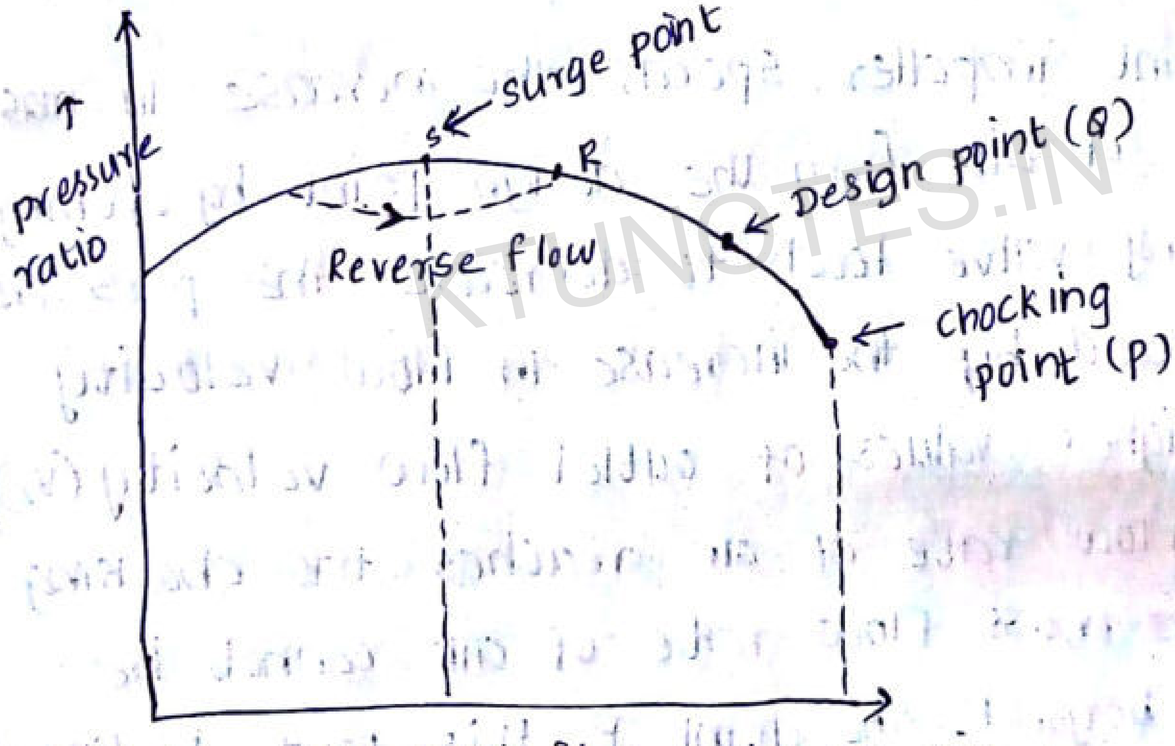
$$\eta_{isen} = \frac{\text{isentropic work input}}{\text{actual work input}}$$

$$= \frac{mc_p\,[T_{02}' - T_{01}]}{mc_p\,[T_{02} - T_{01}]} = \frac{T_{02}' - T_{01}}{T_{02} - T_{01}}$$

$$\therefore T_{02}' - T_{01} = \eta_{isen} \times [T_{02} - T_{01}]$$

$$\therefore \boxed{\text{Pressure coefficient, } \phi_p = \frac{c_p\,\eta_{isen}\,[T_{02} - T_{01}]}{V_{w2}\,u_2}}$$

Surging & Chocking



Surging is defined as the pulsating air flow through the compressor with high frequency. It is caused by unsteady, periodic and reversal flow through the compressor when the compressor has to operate at lower mass flow rate of air

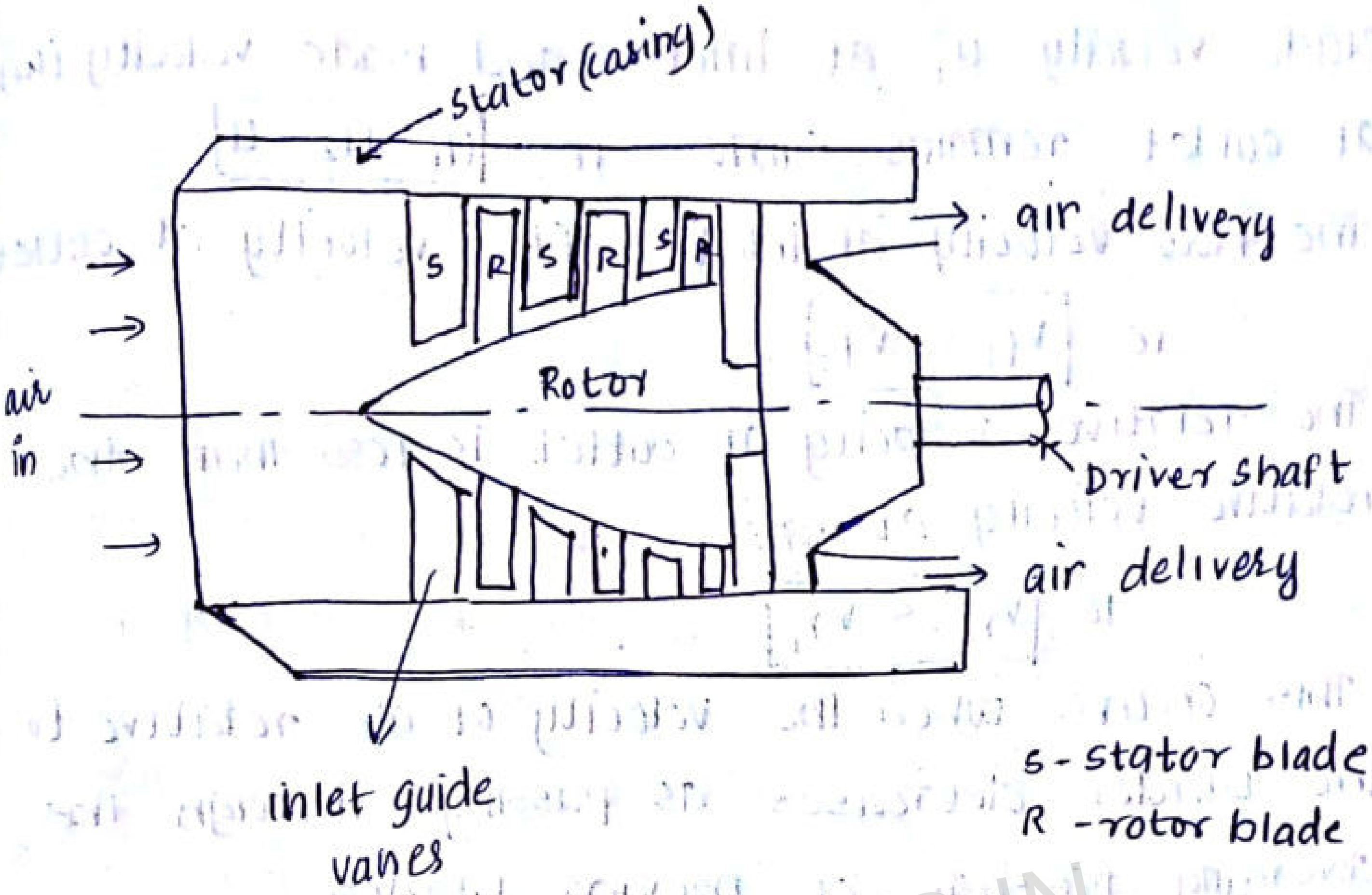
Effects of Surging

1) unstable compressor operations due to reverse flow and pressure oscillations
2) Rising of temp. inside the compressor
3) High frequency vibrations and pressure shocks
4) Mechanical damage which include damage of bearings, blades, casing etc

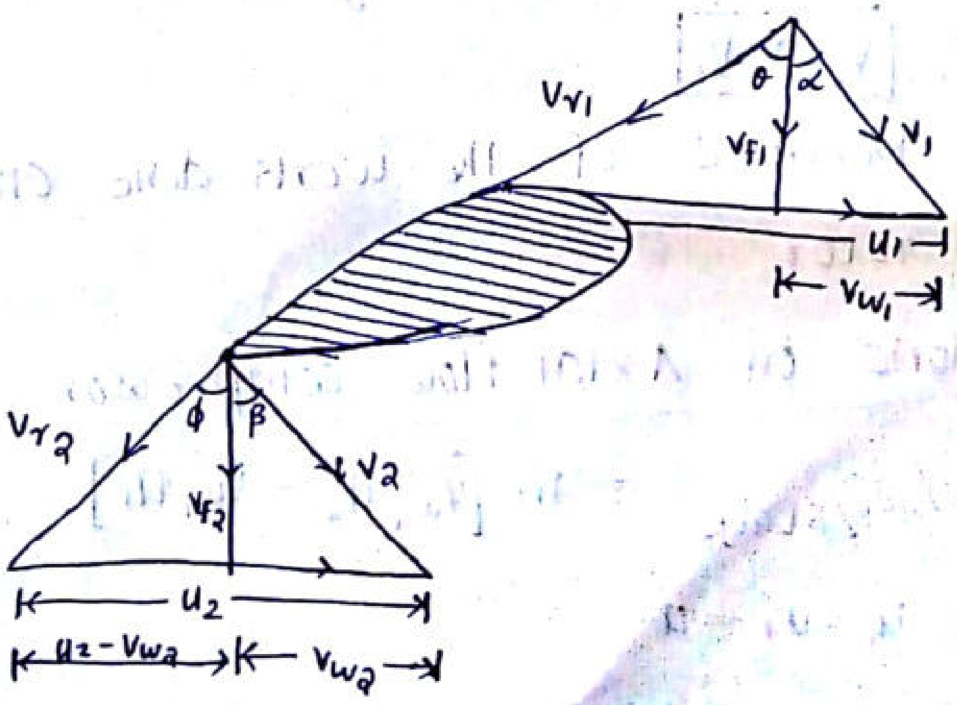
Chocking

At constant impeller speed, the increase in mass flow rate of air from the design point by opening the delivery valve leads to decrease the pressure ratio followed by the increase in flow velocity causing higher values of outlet flow velocity (V_2), the mass flow rate of air reaches the chocking point. The mass flow rate of air cannot be increased beyond the point P. this point is known as chocking point.

Axial flow compressor



Velocity triangles



Design aspects

1) Blade velocity, u_1 at Inlet and blade velocity (u_2) at outlet remains same ; ie ; $\boxed{u_1 = u_2 = u}$
2) The flow velocity at inlet = flow velocity at outlet
   ie, $\boxed{V_{f_1} = V_{f_2}}$
3) The relative velocity at outlet is less than the relative velocity at inlet.
   ie, $\boxed{V_{r_2} < V_{r_1}}$
   This occurs when the velocity of air relative to the blades decreases as passing through the diverging passage of moving blades.
4) The absolute velocity of air at outlet is greater than the absolute velocity of air at inlet
   ie, $\boxed{V_2 > V_1}$
   This is because of the work done on the air by the rotor.

Work done on Axial flow compressor

$$\text{workdone/sec/stage} = m\left[V_{w_2} u_2 - V_{w_1} u_1\right]$$

But $u_1 = u_2 = u$

$$\therefore \text{workdone/sec/stage} = m\,u\left(V_{w_2} - V_{w_1}\right)$$

Total work input/sec $= mu\,(V_{w_2} - V_{w_1})$
$\times$ No. of stages

m = mass rate of flow of air
$= \rho_1 Q_1$
$= \rho_2 Q_2$

From inlet velocity triangle,

$\tan\alpha = \dfrac{V_{w_1}}{V_{f_1}} \quad \therefore V_{w_1} = V_{f_1} \tan\alpha$

From outlet velocity triangle,

$\tan\beta = \dfrac{V_{w_2}}{V_{f_2}} \quad \therefore V_{w_2} = V_{f_2} \tan\beta$

Work done/sec/stage $= mu \cdot (V_{w_2} - V_{w_1})$

$= mu\,(V_{f_2}\tan\beta - V_{f_1}\tan\alpha)$

But $V_{f_1} = V_{f_2}$

$\therefore$ Workdone/sec/stage $= mu \cdot V_{f}\,(\tan\beta - \tan\alpha)$

Total workdone $= mu \cdot V_{f}\,(\tan\beta - \tan\alpha)$
$\times$ No. of stages